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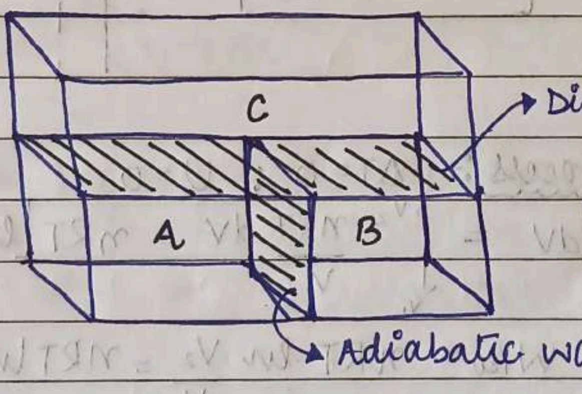
THERMODYNAMICS

#1. Thermodynamics :- branch of science deals with the concepts of heat and other form of energy.

#2. Thermal Equilibrium :- NO flow / transfer of heat or are at same temp.

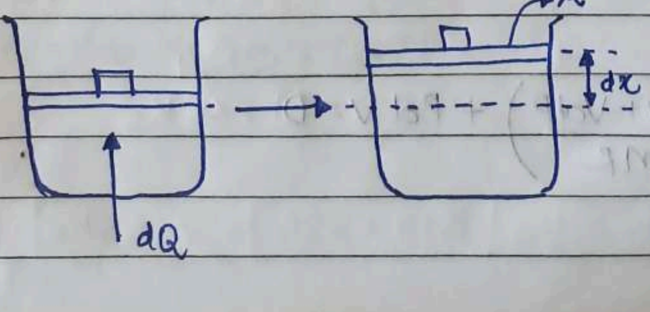
#3 Zeroth Law :- If 2 sys A & B are separately in thermal eqb with a third sys C, then A & B are also in thermal eqb with each other.



$\boxed{T_A = T_B = T_C}$

Volume Expand = Work done by the sys = +ve
Volume Decrease = Work done on the sys = -ve
Heat Supplied to the System = +ve
Heat given out to the System = -ve

#4. First Law :- Conservation of Energy.



Work done $= \vec{F} \cdot \vec{dx}$
$= F \cdot dx$
$= PA\,dx$
$= P\Delta V$

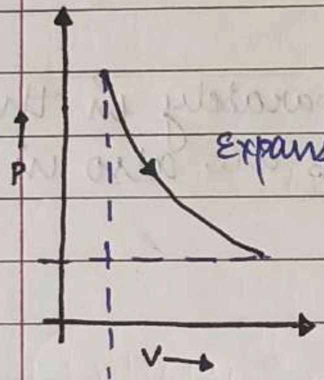
$$\text{Work done} = \int_{V_1}^{V_2} P\,dv$$

$\boxed{dQ = dU \pm dW}$

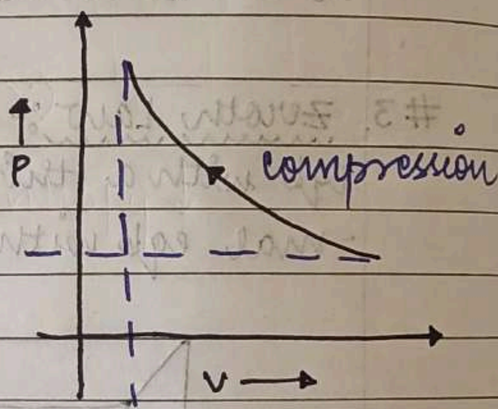
(+ve) work done by the sys

(-ve) work done on the sys

#5. Quasi-static Process :- infinitely slow process such that the system remains in thermal & chemical eqb w surr. (similar but not same)



$\boxed{\text{Area = work done}}$



#6. Isothermal Process :- $\Delta T = 0$; $U = 0$

$$W_{iso} = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = nRT\left[\ln V\right]_{V_1}^{V_2}$$

gas expands $\Delta V > 0$

$$W_{iso} = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2}$$

$\Delta Q = P\Delta V$ gas compress $\Rightarrow \Delta V < 0$

#7. Adiabatic Process :- Heat not passing through, $\Delta Q = 0$

$dQ = dU + dW \Rightarrow dU = C_V dT$ & $dW = PdV$

$$C_V dT + PdV = 0 \quad \text{---(i)}$$

we have $PV = nRT \Rightarrow PdV + VdP = nRdT$

$$\frac{PdV + VdP}{nR} = dT \quad \text{---(ii)}$$

Put (ii) in (i)

$$nC_V\left(\frac{PdV + VdP}{nR}\right) + PdV = 0$$

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$$C_V P dV + C_V V dP + PR dV = 0$$

$$P(C_V + R)\, dV + C_V V dP = 0$$

$$C_P P dV + C_V V dP = 0$$

$\boxed{\text{Divide by } C_V}$

$$\frac{C_P}{C_V} P dV + V dP = 0$$

$$\gamma P dV + V dP = 0$$

$\boxed{\text{Divide by } PV}$

$$\int \frac{\gamma\, dV}{V} + \int \frac{dP}{P} = 0$$

$$P^{1-\gamma} T^{\gamma} = \text{const}$$

$$T V^{\gamma - 1} = \text{const}$$

$$\gamma \log_e V + \log_e P = 0 \Rightarrow \boxed{PV^{\gamma} = K}$$

$$W_{adia} = \int_{V_1}^{V_2} K V^{-\gamma} dV \qquad [PV^{\gamma} = K \Rightarrow P = KV^{-\gamma}]$$

$$= K \left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \qquad [K = P_1 V_1^{\gamma} = P_2 V_2^{\gamma}]$$

$$= \frac{nR}{\gamma - 1}[T_1 - T_2]$$

* work done by the gas $= W_{adia} > 0 \Rightarrow T_2 < T_1$

* work done on the gas $= W_{adia} < 0 \Rightarrow T_2 > T_1$

#8. **Isochoric**

$\Delta V = 0$

$Q = \Delta U \Rightarrow Q = nC_V \Delta T$

Isobaric

$$W = \int_{V_2}^{V_1} P dV = Rn(T_2 - T_1)$$

$$= P(V_2 - V_1)$$

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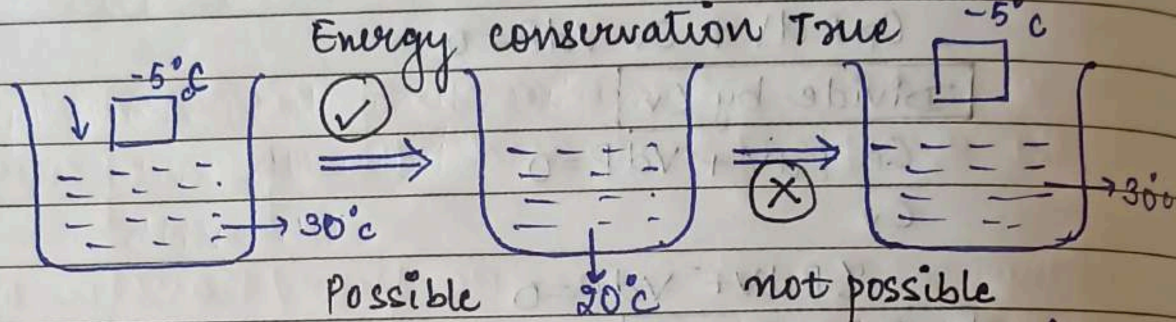
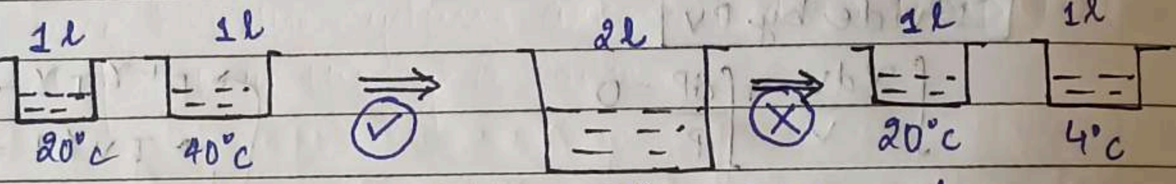
#8. Second Law of Thermo-Dynamics :- Entropy ↑

Energy is conserved but process isnt feasible.

Energy conservation True





Entropy ↑ Entropy ↓

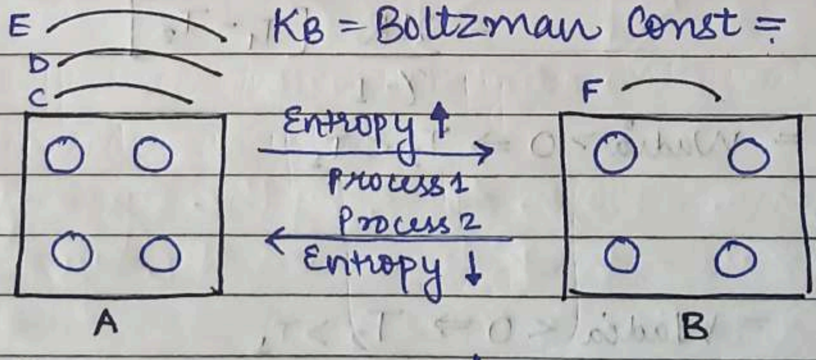
$$\text{Entropy} = S = K_B \log_e W$$

W = no. of thermodynamic microstates

K_B = Boltzman Const = 1.38×10^{23} J/K

W = 4×3×2×1 = 24

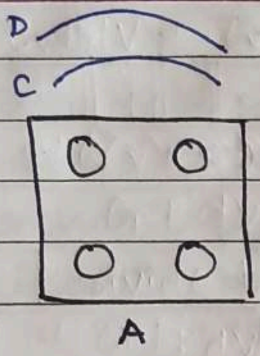
Distribute energy in atoms.

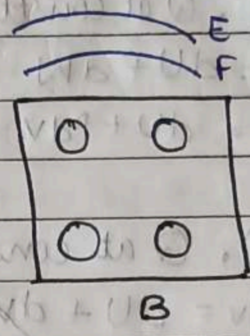


Energy = 3 quanta (Energy is quantised)	1 quanta
$S_A = K_B \ln W$	$S_B = K_B \ln W$
$= K_B \ln 24$	$S_B = K_B \ln 4$
$S_A = K_B\ 3.17$	$S_B = 1.38 K_B$

$$\boxed{S_T = K_B [3.17 + 1.38] = 4.55 K_B}$$

PROCESS = 1



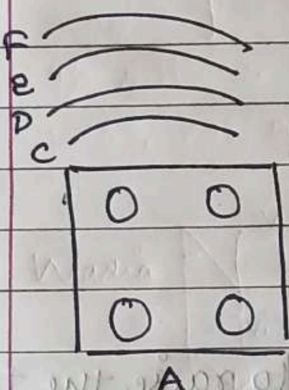


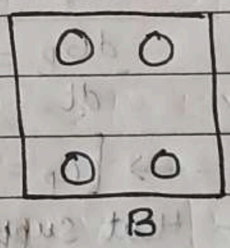
$S_A' = K_B \cancel{12}\ln 12$
$= 2.48\,K_B$

$S_B' = K_B \ln 12$
$= 2.48\,K_B$

$$\boxed{S_T' = 4.96\,K_B}$$

PROCESS = 2





$S''_A = K_B \ln 24$
$= 3.17\,K_B$

$S''_B = K_B \ln 1$
$= 0$

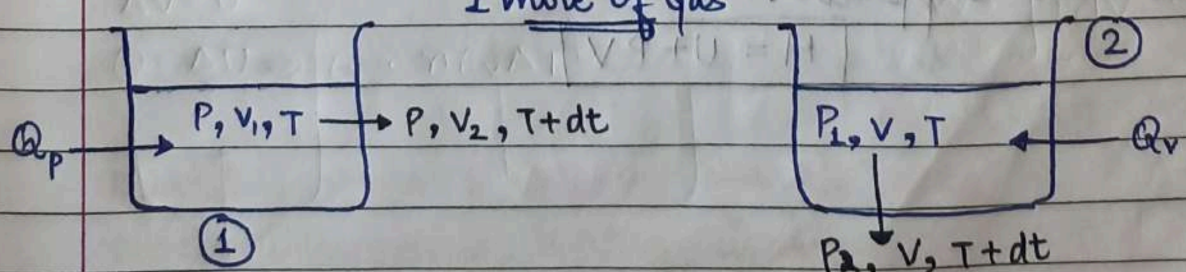
$$\boxed{S_T'' = 3.17\,K_B}$$

" Entropy is a phenomenon considering surrounding "

11 November, 2022

#9. Relation b/w C_p & C_v ⇒ Mayer's Formula

1 mole of Gas.



System ①, Q at constt. pressure.

$$dQ_p = dU + dW$$
$$dQ_p = dU + PdV$$

System ②, Q at constt. volume

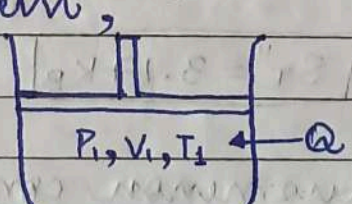
$$dQ_v = dU + \cancel{dW}^{0}$$
$$dQ_v = dU$$

Since both the systems are raised to Temp T+dt, dU for both the sys is same.

$$dQ_p = dQ_v + PdV$$
$$dQ_p = dQ_v + RdT$$
$$\frac{dQ_p}{dt} = \frac{dQ_v}{dt} + R$$
$$\Rightarrow \boxed{C_p = C_v + R}$$

C_p :- Heat supplied to 1 mole of a gas to raise the temp at Const. pressure to 1°.

#10. Difference b/w Enthalpy & Internal energy.
consider a system,



So the Enthalpy is the heat used to bring the system to the configuration P_1, V_1, T_1 and Inc. when heat is supplied to system. Internal energy will be due to heat supplied.

$$\boxed{H = U + PV}$$

THERMO-DYNAMICS

(a) Isothermal process $\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$
(b) Isobaric process $\Rightarrow \Delta P = 0$
(c) Isochoric process $\Rightarrow \Delta V = 0 \Rightarrow W = 0$
} Exchange of Heat possible.
(d) Adiabatic process $\Rightarrow \Delta Q = 0$

\# $dQ = dU + \underline{dW}$
↳ work done by the system

for (a) ; $\boxed{dQ = dW}$
for (c) ; $\boxed{dQ = dU}$
for (d) ; $dU + dW = 0 \Rightarrow \boxed{dU + PdV = 0} \Rightarrow \underset{\text{Decrease}}{dU\downarrow \Rightarrow dT\downarrow}$
$\Rightarrow PdV = -dU = (U_f - U_i)$
$\Rightarrow U_i > U_f$

$U = nC_v\Delta T$
When volume is constant, $W = 0$, $Q \rightarrow \Delta U$
C_v :- heat supplied at const volume to 1 mole to raise 1 unit Temp.
for n moles & ΔT Temp rise, $U = nC_v\Delta T$.

\# THIRD LAW OF THERMODYNAMICS
at $T = 0K \Rightarrow S$ becomes constant & gives its base value.

\# CONTINUM MODEL :- continuous Model :- using assumptions
When we cannot study the behaviour of individual particle, we study it for as a system as whole.

Continuous Model :- Ideal system.

Reversible process :- Entropy change zero
Irreversible process :- Entropy increases.

Q. Prove that Entropy of an Irreversible process always increases.

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MAXWELL

• Thermodynamical Relations •

#1. From first Law of Thermodynamics

$dQ = dU + dW$ (Work done by the system)

$ds = \frac{dQ}{T}$, $dW = PdV$

$Tds = du + PdV$

$$\boxed{dU = Tds - PdV} \quad -(i)$$

(ΔU depends on S & V)
($U = f(S,V)$)

$$\boxed{dU = \left(\frac{\partial u}{\partial s}\right)_V ds + \left(\frac{\partial U}{\partial V}\right)_S dV} \quad -(ii)$$

Since 'U' is an exact differential, Therefore (U is state func^n)

$$\boxed{\left[\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\right)_V\right]_S = \left[\frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\right)_S\right]_V} \quad -(iii)$$

comparing eqns (1) & (2)

$$\left(\frac{\partial U}{\partial S}\right)_V = T \ , \ \left(\frac{\partial U}{\partial V}\right)_S = -P$$

using eq (3)

$$\boxed{\left(\frac{\partial T}{\partial V}\right)_S = \left[-\frac{\partial P}{\partial S}\right]_V}$$ MAXWELL'S ★ FIRST EQUATION.

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#2. $H = U + PV$

$dH = dU + PdV + VdP$

$dH = TdS - PdV + PdV + VdP$ (from(i))

$$\boxed{dH = TdS + VdP} \quad \text{—(iv)}$$

$$\boxed{dH = \left(\frac{\partial H}{\partial S}\right)_P \partial S + \left(\frac{\partial H}{\partial P}\right)_S \partial P} \quad \text{—(v)}$$

comparing (iv) & (v)

$$T = \left(\frac{\partial H}{\partial S}\right)_P \qquad V = \left(\frac{\partial H}{\partial P}\right)_S$$

Since 'H' is exact differential

$$\left[\frac{\partial}{\partial P}\left(\frac{\partial H}{\partial S}\right)_P\right]_S = \left[\frac{\partial}{\partial S}\left(\frac{\partial H}{\partial P}\right)_S\right]_P \quad \text{—(vi)}$$

Putting values in (vi)

$$\boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$$ MAXWELL'S SECOND EQUATIONS

#3. Helmholtz free Energy (F)

$F = U - TS$

$dF = dU - TdS$

$dF = TdS - PdV - TdS - SdT$

$dF = -PdV - SdT$

$$dF = -\left(\frac{\partial F}{\partial V}\right)_T \partial V - \left(\frac{\partial F}{\partial T}\right)_V \partial T$$

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$$\left(\frac{\partial F}{\partial V}\right)_T = P \quad \& \quad \left(\frac{\partial F}{\partial T}\right)_V = S$$

Since 'F' is exact differential

$$\left[\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)_T\right]_V = \left[\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right)_V\right]_T$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$$ MAXWELL'S THIRD EQUATIONS

#4. Gibbs free Energy

$G = H - TS$

$dG = dH - \partial T S - \partial S T$

$dG = T\cancel{dS} + VdP - dTS - \cancel{dS}T$

$dG = VdP - SdT$

$$dG = \left(\frac{\partial G}{\partial P}\right)_T \partial P + \left(\frac{\partial G}{\partial T}\right)_P \partial T$$

$$\left(\frac{\partial G}{\partial P}\right)_T = V \qquad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

$$\left[\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T\right]_P = \left[\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P\right]_T$$

$$\boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T}$$ MAXWELL'S FOURTH EQUATIONS

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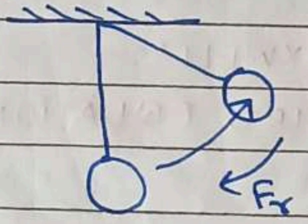
Simple Harmonic motion and Wave Motion

Restoring force :- $F \propto -x$



$$\Rightarrow F = -kx$$

$$\Rightarrow m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{d^2x}{dt^2} + \underbrace{\left[\frac{k}{m}\right]}_{\omega^2} x = 0$$

$$\boxed{0 = \frac{d^2x}{dt^2} + \omega^2 x}$$

Multiplying by $\frac{2dx}{dt}$

$$\frac{2dx}{dt}\frac{d^2x}{dt^2} + 2\frac{dx}{dt}x\omega^2 = 0$$

Integrating

$$\left(\frac{dx}{dt}\right)^2 + x^2\omega^2 = c$$

At $x = a$, $\frac{dx}{dt} = 0$ [dx/dt = velocity which at extreme ($x=a$) = 0]

$$a^2\omega^2 = c$$

$$\left(\frac{dx}{dt}\right)^2 + x^2\omega^2 = a^2\omega^2$$

$$\left(\frac{dx}{dt}\right)^2 = \omega^2\left[a^2 - x^2\right]$$

$$\frac{dx}{dt} = \omega\sqrt{a^2-x^2}$$

$$\frac{dx}{\sqrt{a^2-x^2}} = \omega\, dt$$

Integrating

$$\sin^{-1}\left(\frac{x}{a}\right) = \omega t + \phi \rightarrow \text{constant}$$

$$\frac{x}{a} = \sin(\omega t + \phi)$$

$$\boxed{x = a\sin(\omega t + \phi)}$$

$\phi' = \phi + \pi/2$, then

$$\boxed{x = a\cos(\omega t + \phi')}$$

If a linear diff eq^n have 2 sol^{ns}, its linear combination will also satisfy the eq^n.

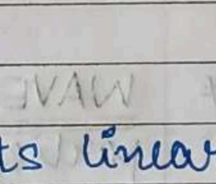
$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(dx/dt\right)^2 = \boxed{\tfrac{1}{2}m\omega^2(a^2-x^2)}$$

$$F = -\frac{dU}{dt} \Rightarrow \cancel{-}Kx = \cancel{-}\frac{dU}{dx}$$

$$= \int Kx\,dx = \int dU$$

Integrating

$$C_1 + \frac{1}{2}Kx^2 = U$$



at $x=0$, $U=0$

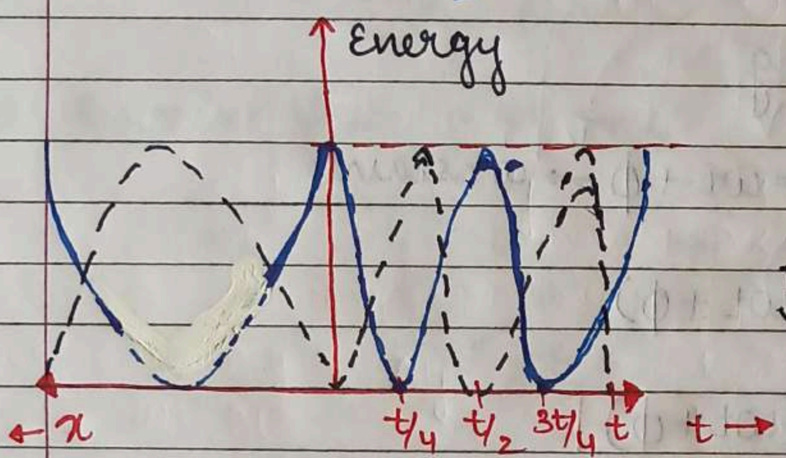
$$\Rightarrow C_1 = 0$$

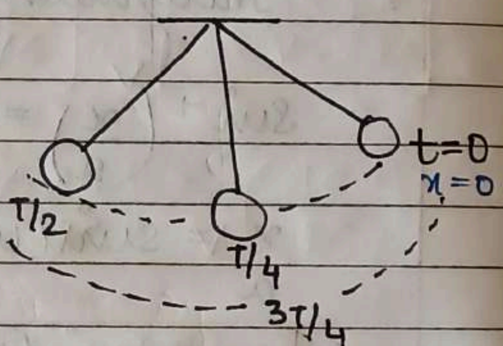
$$\therefore \boxed{U = \tfrac{1}{2}Kx^2}$$

We know $\frac{K}{m} = \omega^2 \Rightarrow U = \frac{1}{2}m\omega^2x^2$

Total Energy :- $\frac{1}{2} m\omega^2 (a^2 - x^2) + \frac{1}{2} m\omega^2 x^2$

$= \boxed{\frac{1}{2} m\omega^2 a^2}$ = constant

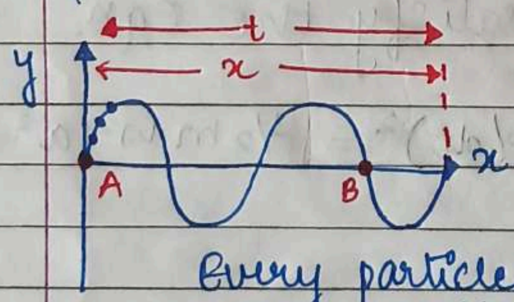




---- KE

—— PE

WAVE :- Transfer of Energy w/o actual movement of particles.



$y = a\sin(\omega t - kx)$

(+ve x direction)

Every particle on wave is executing SHM

Hence $y = a\sin\omega t$

after time t ⇒ particle B will be in same phase as A. $t = x/v$ (v = velocity)

$\therefore y = a\sin(\omega t - \omega(x/v))$

$$\boxed{y = a\sin[\omega t - kx]}$$ *

Eq^n of diff. eq^n of wave.

$k = \frac{\omega}{v}$

$k = \frac{2\pi}{\lambda}$

$T = \frac{2\pi}{\omega}$

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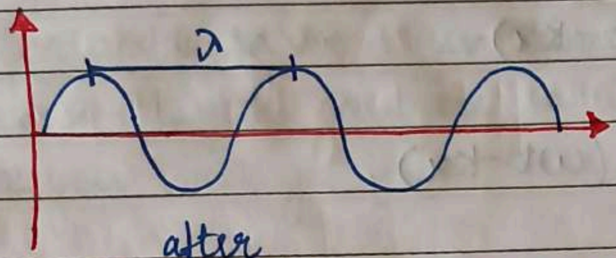
* wave have periodicity both in space (dis) & time.



\# dis after which wave repeat itself = λ = wavelength

⇒ Proof :-

$$x \to x+\lambda$$

$$y' = a\sin(\omega t - k(x+\lambda))$$
$$= a\sin(\omega t - kx - k\lambda)$$
$$y' = a\sin(\omega t - kx - 2\pi) = y$$

sin has periodicity of 2π hence

$$y' = y.$$

\# Time period (T) :- time after which particle repeats its motion.

⇒ Proof :-

$$t \to t+T$$
$$y' = a\sin[\omega(t+T) - k\omega]$$
$$= a\sin[\omega t + 2\pi - k\omega]$$
$$y' = y$$

\# after T, wave covers λ distance.

* Differential Equation of wave.

$$y = A\sin(\omega t - kx)$$

$$\frac{\partial y}{\partial x} = \cancel{A\sin} \, A\cos(\omega t - kx)(-k)$$

$$\frac{\partial^2 y}{\partial x^2} = -A\sin(\omega t - kx)(k^2) = -Ak^2 y$$

Differentiating wrt t

$$\frac{\partial y}{\partial t} = A\omega\cos(\omega t - Kx)$$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - Kx)$$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 y \quad \& \quad \frac{\partial^2 y}{\partial x^2} = -AK^2 y$$

$$\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2} = -Ay \quad \& \quad \frac{1}{K^2}\frac{\partial^2 y}{\partial x^2} = -Ay$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\omega^2}{K^2}\frac{\partial^2 y}{\partial x^2}$$

We know, $V = \frac{\omega}{K}$

$$\boxed{\frac{\partial^2 y}{\partial x^2} \times V^2 = \frac{\partial^2 y}{\partial t^2}}$$

Date :- 28 Nov, 2022

Gradient, Divergence and Curl (point property)

(a) Scalar field :- A function representing a scalar quantity in a desired region which depends upon coordinates & you will get your ans by simple substitution.

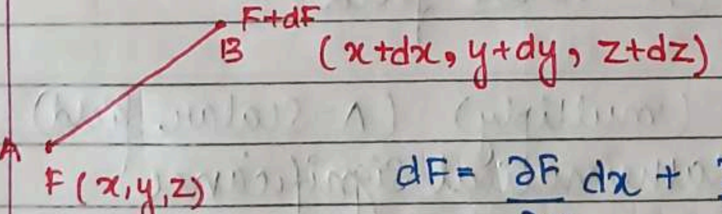
(b) Vector field :- A function which depens on coordi-nates representing a vector field & you will get mag. & direction by substituting coordinates.

(c) Gradient :-

$\nabla \rightarrow$ Del (del operator) $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (cartesian coordinates)

$$\nabla F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)F$$

$$= \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z} \quad (F = \text{Scalar field})$$



$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

$$dF = \left(\hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)F\cdot\left(\partial x\,\hat{i} + \partial y\,\hat{j} + \partial z\,\hat{k}\right)$$

$dF = \nabla F \cdot \vec{dr}$ $\quad \boxed{dr = \partial x\,\hat{i} + \partial y\,\hat{j} + \partial z\,\hat{k}}$

for maximum dF

$$|dF| = |\nabla F|\,|\vec{dr}|\cos\theta$$

for max$|dF|$, $\cos\theta = 1 \Rightarrow \theta = 0$ for const $\vec{dr}$.

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$|dF| = |\nabla F||d\vec{r}|$

Gradient gives the dir of maximum change which is $\perp$ to the surface of point corresponding to that point.

Eg:- $F = xyz$

$\nabla F = \hat{i}yz + \hat{j}xz + \hat{k}xy$

$(x,y,z) = (1,1,1)$

$\nabla F = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$|\nabla F| = \sqrt{1+1+1} = \sqrt{3}$

dir of max change $= \hat{i} + \hat{j} + \hat{k}$

Mag. of " " $= \sqrt{3}$

30 Nov, 2022

Divergence :- $\nabla \cdot \vec{A}$ (A = vector field)

Curl :- $\nabla \times \vec{A}$ (A = vector field)

Gradient :- ∇A (multiply) (A = Scalar field)

Mathematical Significance

DIVERGENCE :-

$\Rightarrow$ $\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

CURL :-

$$\rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \vec{\nabla} \times \vec{A}$$

PHYSICAL SIGNIFICANCE

* **Divergence :-**

divergence is the flux through unit volume in unit time.

$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q_{in}}{\epsilon_0}$ gives flux in arbitary volume

& $\nabla\cdot\vec{E}$ gives flux in unit volume.

$$\nabla\cdot\vec{E} = \lim_{\Delta V\to 0} \frac{\oint \vec{E}\cdot\vec{ds}}{\Delta V} \quad (V\to\text{volume})$$

Flux :- passing through | coming out.

+ve :- outgoing 0 :- no flux -ve :- incoming flux

* **Curl :-** $\nabla\times\vec{E}$

It gives Rotating effect of that vector field at that point.

$$\nabla\times\vec{F} = \lim_{\Delta S\to 0} \frac{\oint_C \vec{F}\cdot\vec{dr}}{\Delta S}$$

circumference

1 Dec, 2022

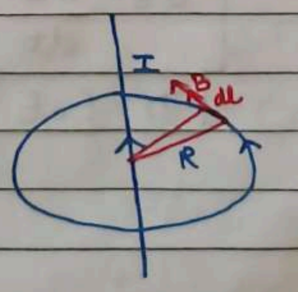
GAUSS LAW

$$\oint \vec{E}\cdot\vec{ds} = \Phi = \frac{q_{in}}{\epsilon_0}$$

Gauss law, proof & applications → Assignment.
(sphere, shell, ∞ long cylinder, ∞ sheet)

MODIFIED AMPERE CIRCUITAL LAW

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$
(line integral

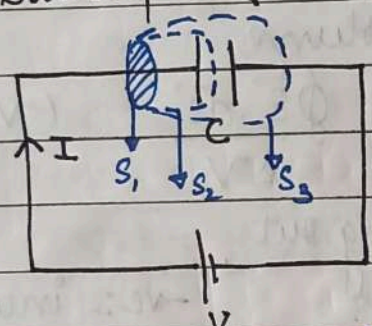


PROOF:- $\oint_c \vec{B}\cdot\vec{dl} = \oint B\,dl\cos\phi \qquad \cos\phi = \cos 0^\circ = 1$

$$= \oint B\,dl = \oint \frac{\mu_0 I}{2\pi R}\,dl$$

$$= \frac{\mu_0 I}{2\pi R}\oint dl = \frac{\mu_0 I}{2\pi R}\times 2\pi R = \mu_0 I$$

But it fails ?



$\oint B\,dl = \mu_0 I_c$ for surface 1
$= 0$ for surface 2
$= \mu_0 I_c$ for surface 3

which cannot happen, hence there should be I_d in b/w capacitors. Hence there should be B also due to varying E.

$$C = \frac{q}{V}$$

$$q = CV = \frac{A\epsilon_0 V}{d}$$

$$q = A\epsilon_0 (E)$$

$$\frac{dq}{dt} = A\epsilon_0 \frac{dE}{dt}$$

$$I_d = A\epsilon_0 \frac{dE}{dt}$$

$$\frac{I_d}{A} = \frac{d}{dt}(\epsilon_0 E)$$

$$\boxed{J_d = \frac{dD}{dt}}$$

$$\boxed{D = \epsilon_0 E}$$ D = displacement vector

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Modified Law :- $\oint \vec{B}\cdot\vec{dl} = \mu_0(I_c + I_d)$ [I_c = Conduction, I_d = Displacement]

for surface 2 $\Rightarrow \oint \vec{B}\cdot\vec{dl} = \mu_0 I_d$

As long as ϵ is changing, I_d comes into picture.

Conductors = conduction current
Insulators = Displacement "
Semi conductors = Diffusion current

MAXWELL'S EQUATION

Date :- 5 Dec, 2022

Integral form	Differential Eqn
(a) $\oint_s \vec{E}\cdot\vec{ds} = \frac{q_{in}}{\epsilon_0}$	$\oint_s \vec{E}\cdot ds = \oint_v (\vec{\nabla}\cdot\vec{E})\, dv = \oint_v \frac{\rho}{\epsilon_0} dv$
$q = \oint_v \rho\, dv$	(using Gauss Divergence thm)
$\rho = \frac{\text{charge}}{\text{volume}}$	$\oint_v \left(\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\epsilon_0}\right) dv = 0$
	$\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\epsilon_0} = 0$
$\boxed{\oint_s \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\oint_v \rho\, dv}$	$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$ → (flux through any volume) [flux through surface enclosing unit volume]
(b) $\oint_s \vec{B}\cdot\vec{ds} = 0$	$\oint_s \vec{B}\cdot ds = \oint (\vec{\nabla}\cdot\vec{B})\, dv = 0$ $\Rightarrow \boxed{\nabla\cdot\vec{B} = 0}$; $\vec{\nabla}\cdot\vec{H} = 0$
(c) $\text{Emf} = -\frac{d\phi}{dt}$ $\left[E.F = \frac{V}{l}\right]$	$\oint \vec{E}\cdot\vec{dl} = \oint (\vec{\nabla}\times\vec{E})\, ds = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot\vec{ds}$
$\boxed{\oint \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot\vec{ds}}$	$\int_s \left(\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t}\right) d\vec{s} = 0$
	Since this is true for any arbitary surface, ∴ The integrand = 0

$\oint \vec{E}\cdot\vec{dl}$ gives work done in moving a charge (unit) in closed loop

$W = \int F\cdot dr$

$= \int Eq\,dr$ [$q = 1C$]

$W = \oint E\,dr$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}}$$

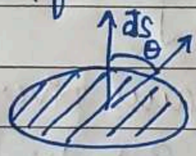
$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$$

[change in B gives rotating E]
Electromotive force works on unit charge to rotate it in E.

(d) $\oint \vec{B}\cdot\vec{dl} = \mu_0[I_c + I_d]$

$= \mu_0\int[\vec{J} + \vec{J_d}]\cdot ds$ $\boxed{I = J\,ds}$

$J = \frac{\text{current}}{\text{Area}}$, J in ds area is J. ds for situation like



$\oint (\vec{\nabla}\times\vec{B})\,ds = \mu_0\int(\vec{J} + \vec{J_d})\,\vec{ds}$

$\int_s (\vec{\nabla}\times\vec{B} - \mu_0(\vec{J} + \vec{J_d}))\,ds = 0$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \vec{J_d})}$$

$\vec{B} = \mu_0\vec{H}$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J_d}}$$

Some results

(i) $\oint_s \vec{A}\cdot\vec{ds} = \oint_v (\vec{\nabla}\cdot\vec{A})\,dv$ Gauss Divergence theorem

(ii) $\oint_c \vec{A}\cdot\vec{dl} = \int_s (\vec{\nabla}\times\vec{A})\,ds$. Strokes threm

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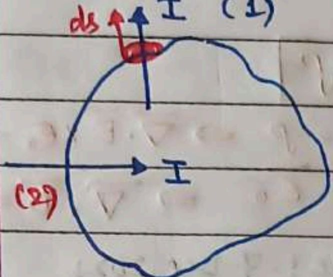
EQUATION OF CONTINUITY

↳ conservation of charge



If I is flowing outside, Q↓ b/c $I = \frac{dq}{qt}$

If I is flowing inside, Q↑

for(1)

$$I = \left|\frac{dQ}{dt}\right| = -\frac{dQ}{dt} \quad (\text{for } 1)$$

for 2

$$I = \oint_s \vec{J}\cdot\vec{ds} = -\frac{d}{dt}\oint_v \rho\, dv$$

$$I = \oint -\vec{J}\cdot\vec{ds} = \frac{dQ}{dt}$$

$$\oint_v (\vec{\nabla}\cdot\vec{J})\, ds = -\oint \frac{d\rho}{dt}\, dV$$

$$\oint_v \left(\vec{\nabla}\cdot\vec{J} + \frac{d\rho}{dt}\right) dV = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$ Eq^n of continuity

Imp ques

Ques. Derive $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$ from $\vec{\nabla}\times\vec{H} = \vec{J}\times\vec{J_d}$

OR

show that eq^n of continuity is contained in fourth Maxwell equation.

$$\nabla\times\vec{H} = \vec{J} + \vec{J_d}$$

Take divergence on both sides

$$\underbrace{\vec{\nabla}\cdot[\vec{\nabla}\times\vec{H}]}_{=0} = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J_d}$$

$$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J_d} = 0$$

$$J_d = \frac{\partial\vec{D}}{\partial t}, \quad \vec{D} = \epsilon\vec{E}$$

$$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\left[\frac{\partial\vec{D}}{\partial t}\right] = 0$$

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$$\vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}\left[\vec{\nabla}\cdot\vec{D}\right] = 0$$

$$\left[\vec{\nabla}\cdot\vec{D} = \rho\right]$$

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

$$\left[\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \vec{\nabla}\cdot\vec{E}\times\epsilon_0 = \rho \Rightarrow \vec{\nabla}\cdot\vec{D} = \rho\right]$$

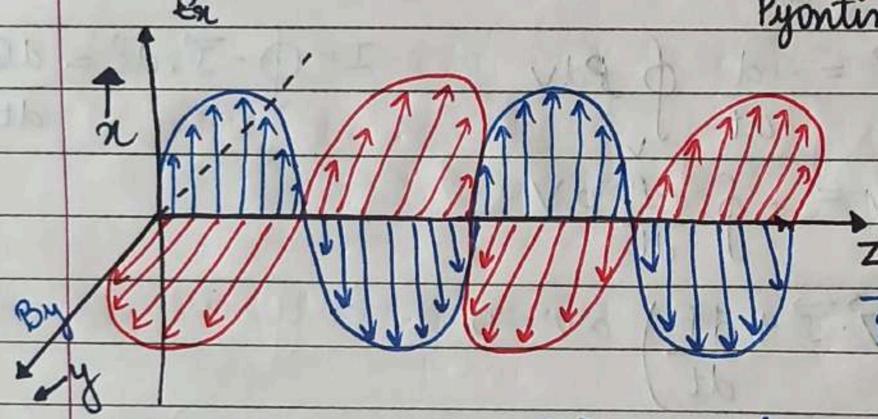
Date :- 7 Dec, 2022

PYONTING VECTOR

$\vec{S}$ — and Pyonting theorem —

Pyonting vector gives Intensity (I) of EM wave

$$I = \frac{\text{Energy}}{At}$$



$$\vec{S} = \vec{E}\times\vec{H} = \frac{\vec{E}\times\vec{B}}{\mu}$$

$$\vec{E_x} = E_0 \sin(\omega t - kz)$$
$$\vec{B_y} = B_0 \sin(\omega t - kz)$$

in vaccum

$$\boxed{\frac{E_0}{B_0} = c}$$

$$E_x \times B_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ E_0\sin(\omega t - kx) & 0 & 0 \\ 0 & B_0\sin(\omega t - kx) & 0 \end{vmatrix}$$

$$= \hat{i}[0] - \hat{j}[0] + \hat{k}\left[E_0 B_0 \sin^2(\omega t - kx)\right]$$

Hence; $\vec{S} = E_0 B_0 \sin^2(\omega t - kz)/\mu$

$$\langle \vec{S} \rangle = \frac{E_0 B_0}{\mu} \langle \sin^2(\omega t - kz) \rangle$$ over a cycle.

$$\boxed{\langle \vec{S} \rangle = \frac{E_0 B_0}{2\mu}}$$ as $\langle \sin^2(\omega t - kz) \rangle = 1/2$ over a cycle.

$$= \frac{1}{T}\int_0^T \sin^2(\omega t - kz)\, dt = \frac{1}{2}$$

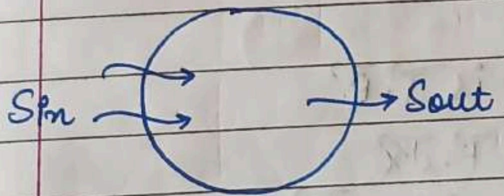
– Pyonting Theorem –

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} = -\frac{\partial \vec{B}}{\partial t} \quad \text{— (i)}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{J_d} \quad \text{— (ii)}$$

$$S_{in} - S_{out} = \nabla \cdot S = -\frac{\partial U}{\partial t} - (J.E) \quad \text{(work done on } \vec{E}\text{)}$$

$$U = \frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2$$



Using (i)

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \text{— (3)}$$

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \vec{J} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad \text{— (4)}$$

$$\left[\nabla \times \vec{H} = \vec{J} + \vec{J_d} = J + \frac{\partial \vec{D}}{\partial t} = J + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

(3) – (4)

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \vec{E} \cdot \vec{J} - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\nabla(\vec{E} \times \vec{H}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} - \vec{J} \cdot \vec{E}$$

$$= -\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \frac{\epsilon}{2} \frac{\partial E^2}{\partial t} - \vec{J} \cdot \vec{E}$$

$$= -\frac{1}{2} \left[\frac{\partial \mu H^2}{\partial t} + \frac{\partial \epsilon E^2}{\partial t} \right] - \vec{J} \cdot \vec{E}$$

$$\nabla(\vec{E} \times \vec{H}) = -\frac{\partial}{\partial t} \left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2} \right] - \vec{J} \cdot \vec{E}$$

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$$\nabla\cdot\vec{S} = -\frac{\partial U}{\partial t} - \vec{J}\cdot\vec{E}$$

Taking volume Integral

$$\int_V (\nabla\cdot\vec{S})\,dv = -\int_V \frac{\partial U}{\partial t}\,dV - \int_V \vec{J}\cdot\vec{E}\,dv$$

$$\int_S S\,ds = -\int \frac{\partial U}{\partial t}\,dv - \int \vec{J}\cdot\vec{E}\,dv$$

why work done is $\vec{J}\cdot\vec{E}$

Work done on charge 'q' by electric field $= \vec{F}\cdot\vec{dx}$

$= \vec{E}q\cdot\vec{dx}$

Work done per unit time $= \vec{E}q\cdot\frac{\vec{dx}}{dt}$

$= \vec{E}q\vec{v}$

why $Nq\vec{v} = \vec{J}$ $\quad = Nq\vec{v}\cdot\vec{E}$

$\vec{J} = Nq\vec{v}$ $\quad = \vec{J}\cdot\vec{E}$

$$I = \frac{dq}{dt} \Rightarrow \frac{I}{A} = \frac{Nq}{tA}\frac{\times x}{\times x} = \frac{Nqv}{A\cdot x} = \frac{Nq\,\text{Velocity}}{\text{volume}}$$

PROPAGATION OF EM WAVES

(a) Non-conducting medium / Dielectric / Air

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = 0$ — (1) (no charge in non-conducting med), $\quad \vec{\nabla}\times\vec{E} = -\mu\frac{\partial H}{\partial t}$ — (3)

$\vec{\nabla}\cdot\vec{H} = 0$ — (2) $\quad \vec{\nabla}\times\vec{H} = \vec{J_c} + \vec{J_d} = 0 + \epsilon\frac{\partial\vec{E}}{\partial t}$ — (4

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Taking curl of eq^n (3)

$$\nabla \times (\nabla \times \vec{E}) = -\mu \frac{\partial}{\partial t} [\nabla \times \vec{H}]$$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left[\epsilon \frac{\partial \vec{E}}{\partial t} \right]$$
$\downarrow$
0

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{or}$$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad - (5)$$

Similarly taking curl of eqn (4) ** (at end)
we will get,

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \left[\begin{array}{l} (\hat{i}\,\partial/\partial x + \hat{j}\,\partial/\partial y + \hat{k}\,\partial/\partial z) \cdot \\ (\hat{i}\,\partial/\partial x + \hat{j}\,\partial/\partial y + \hat{k}\,\partial/\partial z) \end{array} \right.$$

comparing eq^n (5) with standard eq^n of wave.
$\left[\partial^2 y/\partial x^2 = 1/v^2 \; \partial^2 y/\partial t^2 \right]$

$$V^2 = \frac{1}{\mu \epsilon}, \quad V = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{\eta}$$

$\eta \rightarrow$ Refractive index of medium

—x—x—x—x—

$\vec{E} = \vec{E}_0 e^{i[\vec{k} \cdot \vec{r} - \omega t]}$ (sol^n of Eq^n (5))

$\vec{E}_0 = E_{0x} \hat{i} + E_{0y} \hat{j} + E_{0z} \hat{k}$

$\vec{K} = K_x \hat{i} + K_y \hat{j} + K_z \hat{k}$

$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$

Now $\vec{\nabla} \cdot \vec{E} = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left[\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right]$

$$\vec{\nabla} \cdot \vec{E} = \left[\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right] \cdot \left[E_{0x} \hat{i} + E_{0y} \hat{j} + E_{0z} \hat{k} \right] e^{i[K_x x + K_y y + K_z z - \omega t]}$$

$k =$ propogation vector $= \frac{\omega}{v} = \frac{2\pi}{\lambda}$
(only for non - conducting)

$$= E_0 x \frac{\partial}{\partial x}\left[e^{i(k_x x + k_y y + k_z z - \omega t)}\right]$$
$$+ E_0 y \frac{\partial}{\partial y}\left[\quad '' \quad\right]$$
$$+ E_0 z \frac{\partial}{\partial z}\left[\quad '' \quad\right]$$

$$= E_0 x\, i k_x e^{i(\vec{k}\cdot\vec{r} - \omega t)} + E_0 y\, i k_y e^{i(\vec{k}\cdot\vec{r} - \omega t)} + E_0 z\, i k_z e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i\left[\vec{K}\cdot\vec{E_0}\right] e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = i\vec{K}\cdot\vec{E} = 0 \rightarrow \underline{\vec{K}\cdot\vec{E} = 0 \Rightarrow \vec{K}\perp\vec{E}} \quad - (7)$$

$$\nabla \longrightarrow i\vec{k}$$

IIly $\vec{\nabla}\cdot\vec{H} = i\vec{K}\cdot\vec{H} = 0 \Rightarrow \underline{\vec{K}\cdot\vec{H} = 0 \Rightarrow \vec{K}\perp\vec{H}} \quad - (8)$

$\rightarrow$ $\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \epsilon \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$ ** (continued)

$$\vec{\nabla}\cdot(\underbrace{\vec{\nabla}\cdot\vec{H}}_{0}) - \nabla^2\vec{H} = -\epsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{H} = \epsilon\mu \frac{\partial^2 H}{\partial t^2}$$

$$\vec{H} = H_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\frac{\partial \vec{H}}{\partial t} = -i\omega H_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\frac{\partial \vec{H}}{\partial t} = -i\omega\vec{H}$$

$$\nabla\times\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$i(\vec{k}\times\vec{E}) = -\mu(-i\omega\vec{H})$$

$$\vec{K}\times\vec{E} = \mu\omega\vec{H} \Rightarrow \underline{\vec{H}\perp \text{ both } \vec{K} \,\&\, \vec{E}} \quad - (9) \qquad \left\{ \frac{\partial}{\partial t} = -i\omega \right\}$$

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using eqn 7, 8, 9
we can say
~~A.S~~ $(\vec{K}, \vec{E}, \vec{H})$ are mutually perpendicular.

⟹ INTRINSIC IMPEDENCE $(\vec{Z})$ unit $= \Omega$

$$\vec{K} \times \vec{E} = \mu\omega\vec{H}$$

$$|\vec{K}||\vec{E}|\cancel{\hat{n}} = \mu\omega|\vec{H}|\cancel{\hat{n}}$$ [$\hat{n}$ = unit vector along $\vec{H}$]

$$\frac{|\vec{E}|}{|\vec{H}|} = \frac{\mu\omega}{K} = \frac{\mu 2\pi\nu}{2\pi/\lambda} = \mu\nu\lambda$$

$$\vec{Z} = \frac{|\vec{E}|}{|\vec{H}|} = \mu v$$

$$\vec{Z} = \frac{\mu}{\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$$ (for air $\vec{Z} = \sqrt{\frac{\mu_0}{\epsilon_0}}$)

for Air $\vec{Z} = 377\ \Omega$

$\vec{E}$ & $\vec{H}$ are in phase because $\vec{Z}$ is real value

⟹ Pyonting vector

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \vec{E} \times \left[\frac{\vec{k} \times \vec{E}}{\mu\omega}\right]$$

$$= \frac{1}{\mu\omega}\left[(\vec{E}\cdot\vec{E})\vec{k} - \underbrace{(\vec{E}\cdot\vec{k})}_{0}\vec{E}\right]$$

$$= \frac{KE^2\hat{k}}{\mu\omega}$$

$$\langle S \rangle = \frac{2\pi}{\mu(2\pi\nu\lambda)} \langle E^2 \rangle \hat{k}$$

$$\langle S \rangle = \frac{\sqrt{\mu\epsilon}}{\mu} = \sqrt{\frac{\epsilon}{\mu}}\ \frac{E_0^2}{2}$$

$E = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$
$= E_0 \sin(\vec{k}\vec{r} - \omega t)$
$E^2 = E_0^2 \sin^2(\vec{k}\vec{r} - \omega t)$
$\langle E^2 \rangle = \langle E_0^2 \rangle 1/2$

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$$\frac{E_0}{B_0} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\boxed{\langle S\rangle = \frac{BE}{2\mu_0}}$$

Date = 12 Dec, 2022

⇒ PROPAGATION OF EM waves in a conducting Medium.

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0} = 0 \quad \text{— (1)}$$

(charge remains on surface, no charge inside vol)

$$\nabla\cdot\vec{H} = 0 \quad \text{— (2)}$$

$$\nabla\times\vec{E} = -\mu\frac{\partial H}{\partial t} \quad \text{— (3)}$$

$$\nabla\times\vec{H} = \vec{J} + \vec{J_d} = \sigma\vec{E} + \epsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{— (4)}$$

Taking curl of eqn → 3 ($\vec{\nabla}\times$)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \left(-\mu\frac{\partial H}{\partial t}\right) = -\mu\frac{\partial(\vec{\nabla}\times\vec{H})}{\partial t}$$

$$\nabla(\underbrace{\vec{\nabla}\cdot E}_{0}) - \vec{\nabla}^2\vec{E} = -\mu\frac{\partial}{\partial t}\left[\sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}^2\vec{E} = \sigma\mu\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}^2\vec{E} - \sigma\mu\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial\vec{E}}{\partial t} = 0 \quad \rightarrow (5)$$

Solution of equation (5)

$$\vec{E} = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\nabla \rightarrow i\vec{k}$$

$$\nabla\cdot\nabla \rightarrow i\vec{k}\cdot i\vec{k} = -k^2$$

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

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$$-k^2\vec{E} = \mu\sigma(-i\omega)\vec{E} - \mu\epsilon(-i\omega)^2\vec{E} = 0$$

$$\left[-k^2 + \mu\sigma i\omega + \mu\epsilon\omega^2\right]\vec{E} = 0$$

$$\boxed{k^2 = \mu\sigma i\omega + \mu\epsilon\omega^2}$$ (complex value)

$\hookrightarrow$ for non-conducting

$$k^2 = \mu\epsilon\omega^2 \Rightarrow k = \frac{2\pi}{\lambda} \quad **$$

$\downarrow$

for conducting

$$k = \alpha + i\beta$$

$$k^2 = \alpha^2 - \beta^2 + 2\alpha\beta i$$

comparing
$$\left\{\begin{array}{l}\alpha^2 - \beta^2 = \mu\epsilon\omega^2 \\ 2\alpha\beta = \mu\sigma\omega\end{array}\right.$$

solving for α & β

$$(\alpha^2+\beta^2)^2 = (\alpha^2-\beta^2)^2 + (2\alpha\beta)^2$$

$$(\alpha^2+\beta^2)^2 = (\mu\epsilon\omega^2)^2 + (\mu\sigma\omega)^2$$

$$= (\mu\epsilon\omega^2)^2\left[1 + \frac{\mu^2\sigma^2\omega^2}{\mu^2\epsilon^2\omega^4}\right]$$

$$\alpha^2 + \cancel{\beta^2} = \mu\epsilon\omega^2\left[1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right]^{1/2}$$

$+$

$$\alpha^2 - \cancel{\beta^2} = \mu\epsilon\omega^2$$

$$2\alpha^2 = \mu\epsilon\omega^2\left\{\left[1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right]^{1/2} + 1\right\}$$

$$\alpha = \sqrt{\mu\epsilon\omega^2}\left\{\frac{\left[1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right]^{1/2} + 1}{2}\right\}^{1/2}$$

$$\beta = \sqrt{\mu\epsilon\omega^2}\left\{\frac{\left[1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right]^{1/2} - 1}{2}\right\}^{1/2}$$

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$$\vec{E} = E_0 e^{i[\vec{k}\cdot\vec{r} - \omega t]}$$

$$= \vec{E_0} e^{i[k\hat{k}\cdot\vec{r} - \omega t]}$$

$$= \vec{E_0} e^{i[(\alpha + i\beta)\hat{k}\cdot\vec{r} - \omega t]}$$

$$\vec{E} = \vec{E_0} \underbrace{e^{-\beta\hat{k}\cdot\vec{r}}}_{\text{Decreasing exponentially}} e^{i[\alpha\hat{k}\cdot\vec{r} - \omega t]}$$

$v = \frac{\omega}{\alpha}$, $\alpha \rightarrow$ real part of k

$$= \vec{E_0} e^{-\beta r} e^{i[\alpha\hat{k}\cdot\vec{r} - \omega t]}$$

at $r = \frac{1}{\beta} \Rightarrow |\vec{E}| = \frac{E_0}{e}$

$\Rightarrow$ Skin depth $(r = 1/\beta)$
The distance travelled by an em wave in a conducting medium in which its mag. is reduced to $1/e$ to its initial value.

$\Rightarrow$ For good conductor

$$\frac{\sigma}{\epsilon\omega} \gg 1$$

$$\alpha = \sqrt{\mu\epsilon\omega^2}\left[\frac{\left[1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right]^{1/2} + 1}{2}\right]^{1/2}$$

$$\alpha = \sqrt{\mu\epsilon\omega^2}\left[\frac{\sigma}{2\epsilon\omega}\right]^{1/2} = \sqrt{\frac{\mu\epsilon\omega^2\sigma}{2\epsilon\omega}} = \boxed{\sqrt{\frac{\mu\sigma\omega}{2}}}$$

||rly $\boxed{\beta = \sqrt{\frac{\mu\sigma\omega}{2}}}$

⟹ For bad conductors

$\frac{\sigma}{\epsilon\omega} << 1$

$$\boxed{\alpha = \sqrt{\mu\epsilon\omega^2}}$$

$$\beta = \left\{ \frac{\cancel{1} + \frac{1}{2}\frac{\sigma^2}{\epsilon^2\omega^2} - \cancel{1}}{2} \right\}^{1/2} \sqrt{\mu\epsilon\omega^2}$$

$$= \sqrt{\mu\epsilon\omega^2}\left[\frac{\sigma}{2\epsilon\omega}\right]$$

$$\boxed{\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}}$$

⟹ Interinsic Impedence

$\vec{E}$ & $\vec{H}$ are out of phase, $\vec{Z}$ is complex in conducting med.

$$Z = \frac{|\vec{E}|}{|\vec{H}|} = \frac{\mu\omega}{k} \qquad k = \alpha + i\beta \qquad \beta = \text{phase}$$

$$\text{phase diff} = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

Date :- 19 Dec, 2022

NUMERICALS

Q1 The EM wave intensity received on the surface of the earth from the sun is found to be 1.33 KW/m3. Find the amplitude of the electric field vector associated with sunlight.

Q2 A plane EM wave travelling in +z direction in an unbound lossless dielectric with relative permeability $\mu_r = 1$ & relative permittivity $\epsilon_r = 3$ has peak electric field intensity $E_0 = 6\,V/m$. Find the

(a) Speed of wave

(b) Impedence of medium.

(c) Peak Magnetic field intensity (H_0)

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A1 Intensity = 1.33 KW/m²

$$\frac{KW}{m^2} = \frac{J}{S.m^2} = \frac{Energy}{Area.time} = \text{Poyonting vector.}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$\mu = \cancel{1.25\times10^{-6}}\ 4\pi\times10^{-7}$
$\epsilon = 8.85\times10^{-12}$

$$\langle \vec{S} \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0 H_0}{2}$$

$$\boxed{\frac{E_0}{B_0} = C} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\langle \vec{S} \rangle = \frac{E_0 . E_0}{C.2\mu_0} = \frac{E_0^2}{2\mu_0 \times C.}$$

$$\cancel{1.33\times2\times3\times10^8\times1.25\times10^{-6} = E_0^2}$$

$$\cancel{133\times6\times1.25\times10^6\times10^{-6} = E_0^2}$$

$$10^3\times1.33\times4\times\pi\times10^{-7}\times2\times3\times10^8 = E_0^2$$

$$E_0 = 10^3 V/m$$

Q2. $\mu_r = 1$; $\epsilon_r = 3$

(a) $$V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{C}{\sqrt{\mu_r\epsilon_r}} = \frac{3\times10^8}{\sqrt{1.3}} m/s$$
$$= \sqrt{3}\times10^8 m/s$$

(b) $$Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{4\pi\times10^{-7}\times1}{8.85\times10^{-12}\times3}}$$
$$= Z_0\sqrt{\frac{1}{3}} = \frac{377}{\sqrt{3}}\Omega$$

$$\left(Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega \right)$$

(c) $|\vec{H_0}| = \dfrac{|\vec{E_0}|}{Z}$

10^{-7} **Q3.** Show that force $F = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$ is conservative.

A3. conservative = work done in closed path = 0

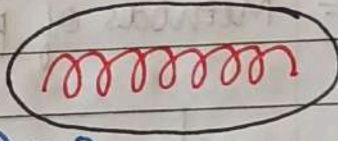
$\Rightarrow$ curl = 0

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz)\right]\hat{i} - \hat{j}\left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right] + \hat{k}\left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz)\right]$$

$= 0$

Hence proved.

Q4. $A = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal find a.



flux = 0

A4. for solenoidal, $\nabla \cdot \vec{A} = 0$

$$\frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+az) = 0$$

$1 + 1 + a = 0$

$\boxed{a = -2}$

Q5. find skin depth for copper.

$\nu = 1\text{MHz}$

$\sigma = 5.8 \times 10^{7}\ \text{S/m}$

$\mu = 4\pi \times 10^{7}$

$\beta = \sqrt{\dfrac{\mu\omega\sigma}{2}}$ $\qquad \omega = 2\pi\nu$

$$\gamma = \frac{1}{\beta} = \sqrt{\frac{2}{\mu\omega\sigma}} = \sqrt{\frac{2}{4\pi\times10^{-7}\times 2\pi\times1\times10^{6}\times5.8\times10^{7}}}$$

$= \cancel{4.5}$

20 Dec, 2022

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Summary :-

\# Methods of producing EF

Static Charge → Gauss law → EF starts +ve, End -ve → $\oint \vec{E} \cdot ds = \frac{q_{in}}{\epsilon_0}$

Time varying MF → Faraday's law → Circulating EF (continuous) (conservative EF) → $\epsilon = \frac{-\partial \phi}{\partial t} = \frac{-\partial}{\partial t} \int_s \vec{B} \cdot \vec{ds}$

\# Methods of producing MF.

Moving charges or current → Biot Savarts law → $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{dL} \times \hat{r}}{r^2}$

varying EF → Modified Ampere circuital law → $\oint \vec{B} \cdot \vec{dl} = \mu_0 (\vec{I_c} + \vec{I_d})$

\# Maxwell equations

(A) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (free charge inside Volume)

(B) $\nabla \cdot \vec{H} = 0$

(C) $\nabla \times \vec{E} = -\mu \frac{\partial H}{\partial t}$

(D) $\nabla \times \vec{H} = \vec{J} + \vec{J_d} = \sigma \vec{E} + \epsilon \dfrac{\partial \vec{E}}{\partial t}$

$\downarrow$

Due to free charges.

→ μ = permeability $= 4\pi \times 10^{-7}$

→ ϵ = permitivity $= 8.85 \times 10^{-12}$

→ $y = A\sin(\omega t - kx)$ Dir. of propagation = $+x$ dir

→ wave = varying in space & time = periodicity in space & time.

→ T time → λ dis covers.

→ Z = Real In phase

→ Z = complex In diff. phase.

PRESSURE & MOMENTUM OF EM WAVES :-

$\langle S \rangle$ = Energy flowing per unit area per unit time = Intensity

$E = \sqrt{p^2c^2 + m_0^2C^4}$

for photon, rest mass $(m_0) = 0$

$E = pc \Rightarrow \dfrac{E}{c} = p = \dfrac{\langle S \rangle}{c}$ = ~~moten~~ $\dfrac{\text{momentum}}{\text{area} \times \text{time}}$

$\dfrac{\langle S \rangle}{c} = \dfrac{\text{Force}}{\text{area}}$ = Pressure $\qquad F = \dfrac{dP}{dt}$

$\dfrac{I}{c}$ = Pressure

Reflecting Surface: Pressure $= \dfrac{2I}{c}$

Absorbing Surface: Pressure $= \dfrac{I}{c}$

Date :- 21 Dec 202

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INTERFERENCE

Interference is Superposition of 2 or more waves resulting in redistribution of energy.

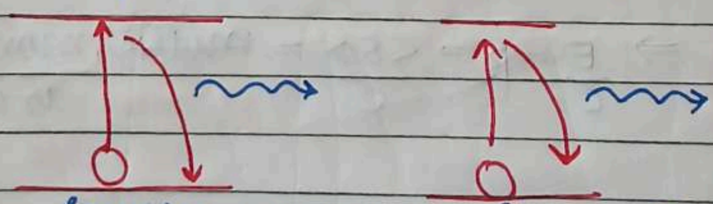
Constructive Interference	Distructive Interference
$I_W > I_1 + I_2$	$I_W < I_1 + I_2$

CONDITIONS :-

(A) COHERANT Sources :- frequency & Amplitude & phase are same.
(B) frequency same ; (E) sources should be close to each other.
(C) Mono chromatic ; (F) Same state of Polarisation on beams Interfering.
(D) Nearly same amplitude, etc.

coherent Sources :- They should be in phase or have same phase difference.

2 Indepent sources can be coherent for same frequency sources for atmost 10^{-8} sec.



Source 1 Source 2

life time of excited atom $= 10^{-8}$ sec
∴ we cannot obtain a sustained pattern

How to make coherent source :-

(1) Division of wavefront.
(2) Division of amplitude. (By reflection & refraction)
By changing medium.

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GE

-: UNIT-3 :-

Diffraction & Polarization

-: HW = YDSE :-

WAVEFRONT :-

locus of points in same phase.

Intensity $\propto$ Amplitude^2

$Y_1 = a_1 \sin \omega t$ $\quad$ $Y_2 = a_2 \sin(\omega t + \delta)$

By principle of Superposition :-

Superposition (Resultant is sum of vector addition of each individual wave)

$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$
$$= a_1 \sin\omega t + a_2 [\sin\omega t \cos\delta + \cos\omega t \sin\delta]$$
$$= (a_1 + a_2 \cos\delta) \sin\omega t + a_2 \sin\delta \cos\omega t$$

g.

Let $a_1 + a_2 \cos\delta = A\cos\phi$ —(i)

$a_2 \sin\delta = A\sin\phi$ —(ii)

i÷ii	$\tan\phi = \dfrac{a_2 \sin\delta}{a_1 + a_2\cos\delta}$

$$y = A\cos\phi \sin\omega t + A\sin\phi \cos\omega t$$
$$y = A \sin(\omega t + \phi)$$

Amplitude :- Squaring and adding (i) & (ii)

$$A^2 = (a_1 + a_2 \cos\delta)^2 + a_2^2 \sin^2\delta$$
$$A^2 = a_1^2 + a_2^2 \cos^2\delta + 2a_1a_2\cos\delta + a_2^2 \sin^2\delta$$
$$\boxed{A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta} \text{ —(iii)}$$

a_1 & a_2 are constt.

constructive :-

for max Amplitude

$\cos\delta = +1$

$\delta = 0, 2\pi, 4\pi, \ldots, 2n\pi$

$n = 0, 1, 2, \ldots$

Destructive :-

for min Amplitude

$\cos\delta = -1$

$\delta = \pi, 3\pi, 5\pi, \ldots (2n+1)\pi$

$n = 0, 1, 2, \ldots$

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Phase diff $= \frac{2\pi}{\lambda} \times$ path difference

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x}$$

Constructive
path diff =
$0, \lambda, 2\lambda, 3\lambda, n\lambda$

Destructive
path diff =
$\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$
$(2n+1)\lambda/2$

Intensity

$$I \propto A^2$$

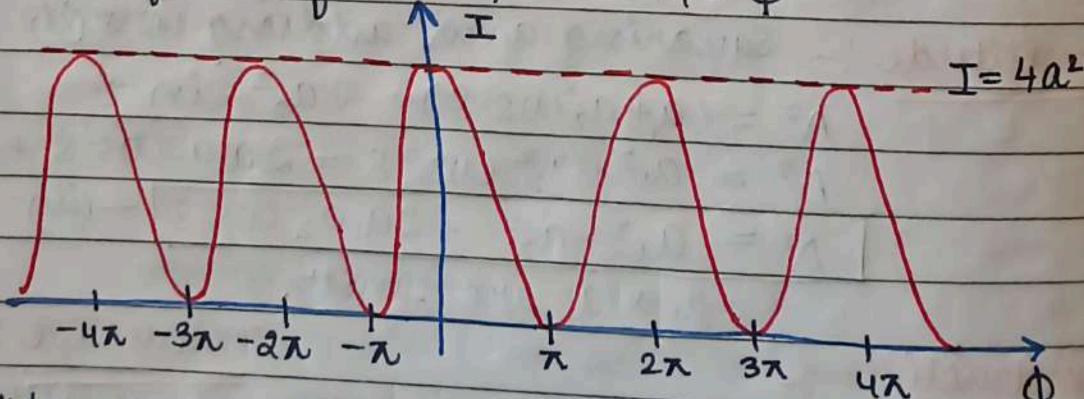
using (iii)

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$
$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta$$

Constructive
$\cos\delta = 1$
$A^2 = (a_1+a_2)^2$
$I = (a_1+a_2)^2$
$I = (\sqrt{I_1}+\sqrt{I_2})^2$

Destructive
$\cos\delta = -1$
$A^2 = (a_1-a_2)^2$
$I = (a_1-a_2)^2$
$I = (\sqrt{I_1}-\sqrt{I_2})^2$

Graph of Interference for I v/s ϕ



HW.

YOUNG'S DOUBLE SLIT EXPERIMENT

Constructive

(a) $\Delta = 0$ (const) Central Bright

$I = 4I_0$

(b) $\Delta = \lambda$ I^{st} Bright

$I = 4I_0$

Destructive

(a) $\Delta = \lambda/2$ I^{st} dark

$I = 0$

path diff $= \frac{dx}{D}$ d = dis b/w slits

D = dis b/w screen & slit.

BRIGHT FRINGE

Path diff $= N\lambda$

$$\frac{dx}{D} = N\lambda$$

$$x = \frac{N\lambda D}{d} \longrightarrow (*)$$

$N=0 \Rightarrow x=0$ Central Bright

$N=1 \Rightarrow x = \frac{\lambda D}{d}$ first Bright

$N=2 \Rightarrow \frac{2\lambda D}{d} = x$ second Bright

DARK FRINGE

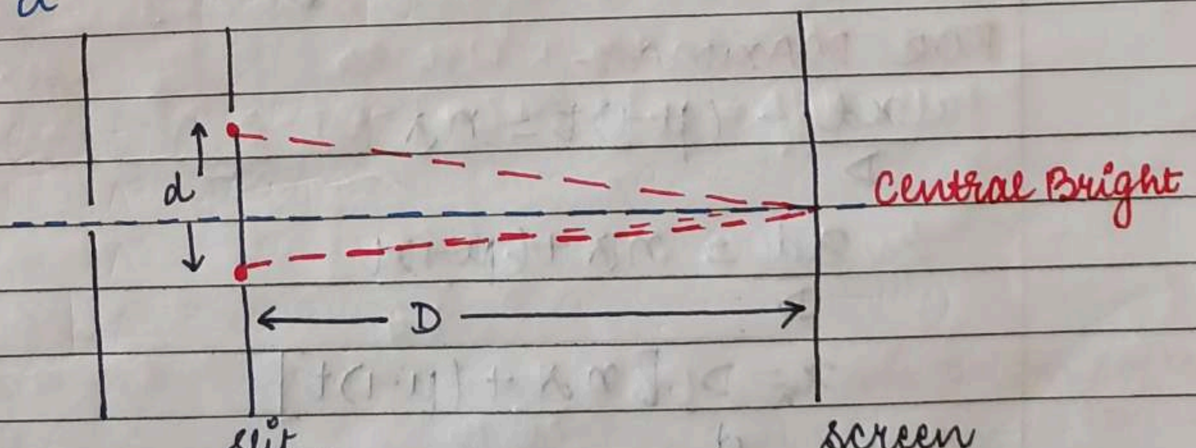
Path diff $= \frac{(2N+1)\lambda}{2}$

$$\frac{dx}{D} = \frac{(2N+1)\lambda}{2}$$

$$x = \frac{(2N+1)\lambda D}{2d}$$

$N=0 \Rightarrow x = \frac{\lambda D}{2d}$ 1^{st} Dark

$N=1 \Rightarrow x = \frac{3\lambda D}{2d}$ 2^{nd} Dark

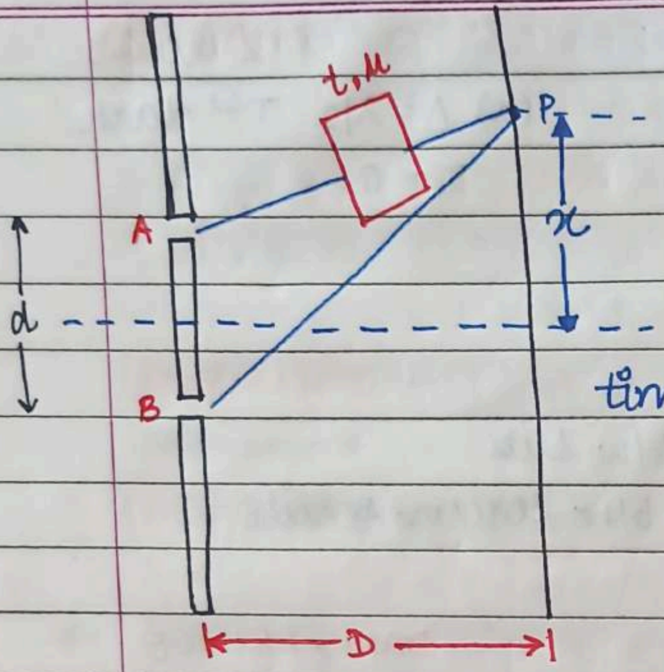


Date :- 22 Dec, 2022

Division of Wavefront :-

Young's Double Slit Experiment

& Fresnel biprism.



Path difference

$$BP - AP = \frac{xd}{D} \quad -(i)$$

fringe width $= \beta = \frac{\lambda D}{d}$

time ~~Path diff~~ $= \frac{AP - t}{C} + \frac{t}{C/\mu}$

$$= \frac{AP - t + \mu t}{C} = \frac{AP + (\mu-1)t}{C}$$

Before Slab = time = $\frac{AP}{C}$

Hence $(\mu-1)t$ is exta optical path intodLuced by Glass Slab

New path diff (after slab) $= BP - [AP + (\mu-1)t]$

$$= \underbrace{BP - AP}_{\text{from (i)}} - (\mu-1)t$$

New path diff $= \frac{xd}{D} - (\mu-1)t$

FOR MAXIMA :-

$$\frac{xd}{D} - (\mu-1)t = n\lambda$$

$$\frac{xd}{D} = n\lambda + (\mu-1)t$$

$$x_n = \frac{D}{d}[n\lambda + (\mu-1)t]$$

from ⊛ $\;\to$ before slab

$$x_n = x + (\mu-1)t\left(\frac{D}{d}\right)$$

Shift $= \frac{D}{d}(\mu-1)t$

Note :- There will be no change in fringe width.

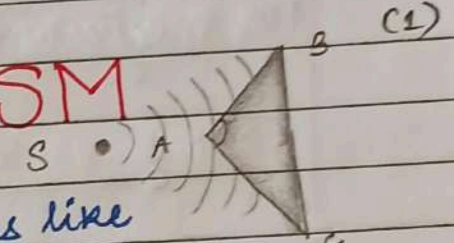
Proof :-

$$x_1 = \frac{D}{d}[\lambda + (\mu-1)t] \qquad x_2 = \frac{D}{d}[2\lambda + (\mu-1)t]$$

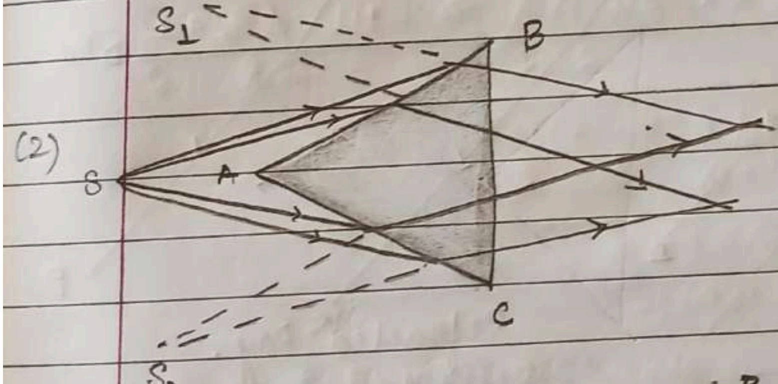
$$x_2 - x_1 = \beta = \frac{\lambda D}{d}$$

FRESNEL BIPRISM

(1)

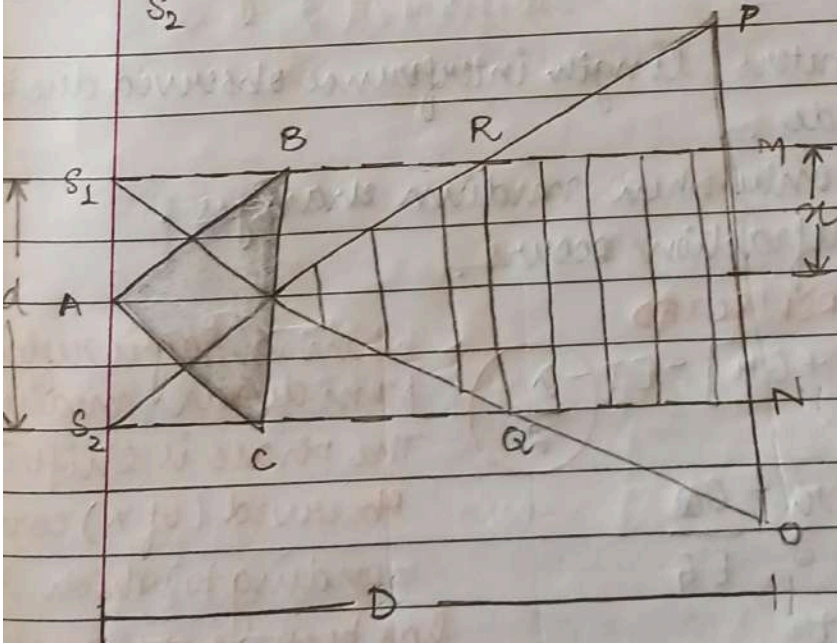


(2)



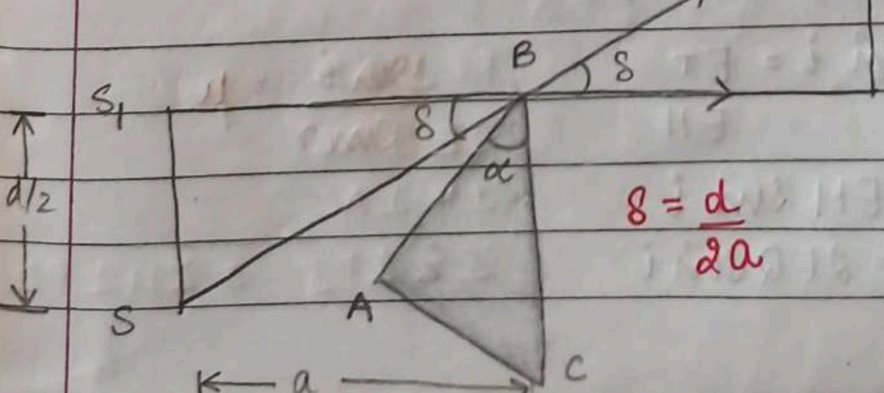
Appears like coming from virtude source S_1 & S_2

$\angle A = 179°$
$\angle B = 30'$
$\angle C = 30'$



Rays from S_1 are confined in S_1 MO & from S_2 are confined in S_2 PN. Results are same as that of YDSE.

Method of finding d



$\delta = \frac{d}{2a}$

Thin prism ∠ of Dev. Deviation

$$\delta = (\mu-1)\alpha$$

$$\frac{d}{2a} = (\mu-1)\alpha$$

$$\boxed{d = 2a(\mu-1)\alpha}$$

Date :- 26 DEC, 2022

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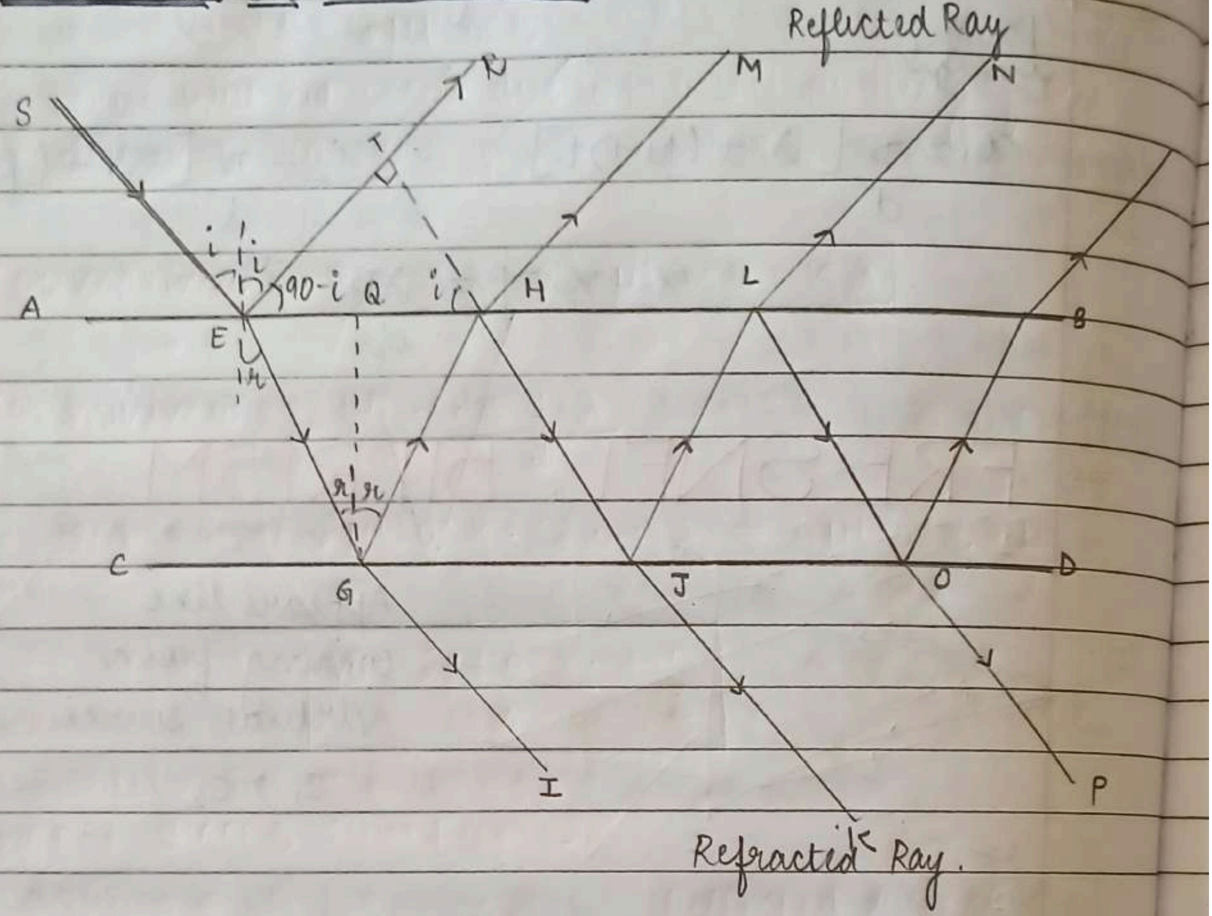
DIVISION OF AMPLITUDE :- Parallel thin Film.

Intensity



Thin film in micrometers, length interference observed due to division of amplitude.

Amplitude divides only when medium changes.

Both reflection & refraction occurs.

FOR REFLECTED

$$\text{Path diff} = \mu[EG + GH] - ET - \frac{\lambda}{2}$$

When a ray is reflected from denser medium. The phase is shift is observed (of π) corresp-onding to which $\lambda/2$ has been subtracted.

In ΔEGQ, $\cos r = \frac{GQ}{EG}$

$$EG = \frac{t}{\cos r}$$

In ΔETH $\sin i = \frac{ET}{EH}$

$$\frac{\sin i}{\sin r} = \mu$$

$$ET = EH \sin i$$

$$ET = 2EQ \sin i$$

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In ΔEGQ $\tan r = \frac{EQ}{GQ} \Rightarrow EQ = t\tan r$

$$PD = 2\mu EG - ET - \frac{\lambda}{2}$$

$$= \frac{2\mu t}{\cos r} - 2t\tan r \sin i - \frac{\lambda}{2}$$

$$= \frac{2\sin i}{\sin r}\frac{t}{\cos r} - 2\mu t \tan r \sin r - \frac{\lambda}{2}$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} - \frac{\lambda}{2}$$

$$PD = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right] - \frac{\lambda}{2}$$

$$PD = 2\mu t\cos r - \frac{\lambda}{2}$$

For maxima $PD = n\lambda$

$$2\mu t\cos r - \frac{\lambda}{2} = n\lambda$$

$$\boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2 ..}$$

For minina

$$PD = (2n-1)\frac{\lambda}{2} \quad n = 1, 2, 3 \quad (PD = \text{Path diff})$$

$$2\mu t\cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{n\lambda = 2\mu t\cos r}$$

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FOR transmitted / Refracted System

$$PD = 2\mu t\cos r$$

$\frac{\lambda}{2}$ term will not occur due to waves travelling from denser to rarer.

For maxima :- $PD = n\lambda \Rightarrow \boxed{2\mu t\cos r = n\lambda}$

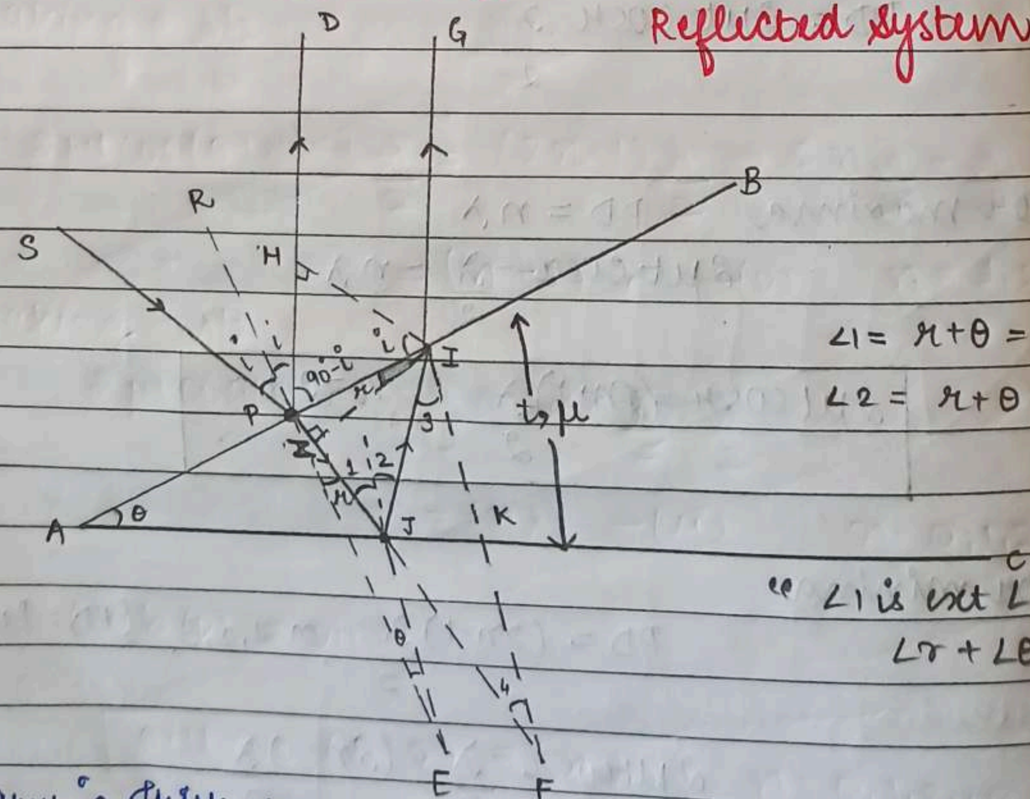
For minima :- $PD = \frac{(2n-1)\lambda}{2}$

$$2\mu t\cos r = \frac{(2n-1)\lambda}{2} \Rightarrow \frac{4\mu t\cos r}{2\lambda} + \frac{1}{2} = n$$

$$\boxed{n = \frac{2\mu t\cos r}{\lambda} + \frac{1}{2}}$$

WEDGE SHAPED THIN FILM:-

Reflected System



$\angle 1 = r + \theta = \angle 3 = \angle 4$

$\angle 2 = r + \theta$

" $\angle 1$ is ext $\angle$ of $\angle r + \angle \theta$. "

varying thickness

$RE \perp AB$

$JE \perp AC$

$IK \perp AC$

$IZ \perp PJ$ & $IH \perp PD$

Path difference b/w PD & IG $= \mu [PJ + JI] - PH \pm \frac{\lambda}{2}$

$= \mu [PZ + ZJ + JI] - PI \pm \frac{\lambda}{2}$

$\mu = \frac{\sin i}{\sin r} = \frac{PH / PI}{PZ / PI} = \frac{PH}{PZ}$

$\mu PZ = PH$

Path Diff $= \mu [ZJ + JI] \pm \frac{\lambda}{2}$ → There is Phase diff of π in incident & reflected ray corresponding to which $\lambda/2$ has been added.

consider ΔIKJ & ΔKJF

$IK = KF$ (by const)

$JK = JK$ (common)

$\angle IKJ = \angle FKJ = 90^\circ$

$\Delta IKJ \cong \Delta FKJ$

$JI = JF$ (by cpct)

Path diff $= \mu [ZJ + JF] \pm \frac{\lambda}{2}$

$= \mu (ZF) \pm \frac{\lambda}{2}$

In ΔZIF

$\frac{ZF}{IF} = \frac{B}{H} = \cos(r+\theta)$

$ZF = 2t \cos(r+\theta)$ $\{IF = 2t\}$

$PD = 2\mu t \cos(r+\theta) \pm \frac{\lambda}{2}$

For maxima :- $2\mu t \cos(r+\theta) \pm \frac{\lambda}{2} = n\lambda$ — (i)

For minima :- $2\mu t \cos(r+\theta) \pm \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$ — (ii)

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Solving (i) "for Maxima"
taking $+\lambda/2$

$$2\mu t\cos(\theta+r) + \frac{\lambda}{2} = n\lambda = (2n-1)\frac{\lambda}{2} \quad n=1,2,3 \rightarrow (iii)$$

taking $-\lambda/2$

$$2\mu t\cos(\theta+r) - \frac{\lambda}{2} = n\lambda = (2n+1)\frac{\lambda}{2} \quad n=0,1,2 \quad -(iv)$$

club (iii) & (iv)

$$\boxed{2\mu t\cos(\theta+r) = (2n-1)\frac{\lambda}{2}} \quad ; n=1,2,3,\ldots$$

Solving (ii) "for minima"

taking $+\lambda/2$

$$2\mu t\cos(\theta+r) + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2} \Rightarrow \cancel{2}n\frac{\lambda}{\cancel{2}} - \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$= (n-1)\lambda \quad -(v) \quad n = 1,2,3,\ldots$$

taking $-\lambda/2$

$$2\mu t\cos(\theta+r) - \frac{\lambda}{2} = \cancel{2}n\frac{\lambda}{\cancel{2}} - \frac{\lambda}{2}$$

$$= (n\lambda) \quad -(vi) \quad n=0,1,2,\ldots$$

club (v) & (vi)

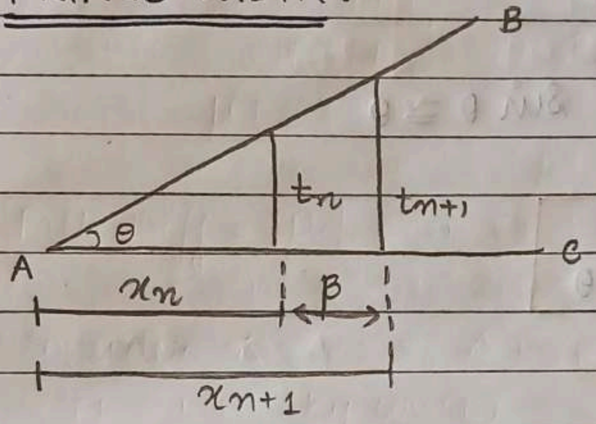
$$\boxed{2\mu t\cos(\theta+r) = (n\lambda)} \quad n=0,1,2,\ldots$$

for transmitted system

Maxima = $2\mu t\cos(\theta+r) = n\lambda$ ($\pm\lambda/2$ discarded)

Minima = $2\mu t\cos(\theta+r) = (2n-1)\frac{\lambda}{2}$

FRINGE WIDTH :-



$\tan\theta = \frac{t_n}{x_n} = \frac{t_{n+1}}{x_{n+1}}$

$\beta = x_{n+1} - x_n$

For maxima :- (n^{th} Maxima)

$$2\mu t_n\cos(r+\theta) = (2n-1)\frac{\lambda}{2} \quad ; n = 1, 2, 3$$

$$2\mu\tan\theta\, x_n\cos(r+\theta) = (2n-1)\frac{\lambda}{2} \rightarrow (i)$$

for $n+1^{th}$ Maxima

$$2\mu x_{n+1}\tan\theta\cos(r+\theta) = (2n+1)\frac{\lambda}{2} \quad -(ii)$$

(ii) - (i)

$$2\mu\cos(r+\theta)\left[x_{n+1} - x_n\right]\tan\theta = \lambda$$

$$2\mu\cos(r+\theta)\beta\tan\theta = \lambda$$

$$\boxed{\beta = \frac{\lambda}{2\mu\cos(r+\theta)\tan\theta}}$$

If light is falling normally :- $r = 0$

$$\beta = \frac{\lambda}{2\mu \tan\theta \cos\theta}$$

$$\beta = \frac{\lambda}{2\mu \sin\theta}$$

for small θ, $\sin\theta \cong \theta$

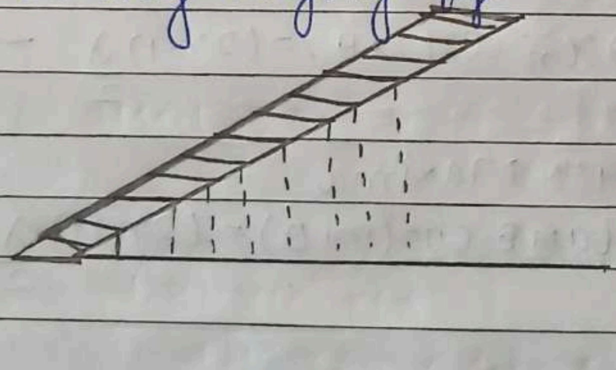
$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Shape of fringes :-

Locus of pts having same thickness

$PD = 2\mu t \cos(\theta + \delta) \pm \frac{\lambda}{2}$

t is only varying qty.



Date = 2 Jan. 2023

NUMERICALS OF EM WAVES.

Q1. Show that in a given vol, the energy of an EM wave is equally shared b/w electric & magnetic field.

Q2. When the amplitude of magnetic field in a plane wave is 2 A/m

(a) determine the ~~magnetic field~~ magnitude of EF for place wave in free space.

(b) determine the magnitude of EF when the wave propagates in a medium which is characterised by $\sigma = 0$, $\mu = \mu_0$, $\epsilon = 4\epsilon_0$

Q3 If avg dis b/w the sun & earth is 1.5×10^{11} m & power radiated by sun is 3.8×10^{26} W. Show that average solar energy incident on earth surface is 2 cal / cm². ~~mm~~ min.

A1. $\nabla \cdot \vec{S} = -\frac{\partial U}{\partial t} - J \cdot E$; U = Total Energy.

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2$$

$$Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad -(i)$$

To prove :- $1 = \frac{\frac{1}{2}\epsilon_0 E^2}{\frac{1}{2}\mu_0 H^2} \Rightarrow \frac{\frac{1}{2}\epsilon_0}{\frac{1}{2}\mu_0}\frac{\mu_0}{\epsilon_0} = 1$ using (i)

A2. (a) A/m = Ampere per meter. (unit of H (Intensity of magnetisation)).

$$\frac{E_0}{B_0} = C \Rightarrow E_0 = CB_0 = 3 \times 10^8 \times 2 \times \mu = 3 \times 2 \times 4\pi \times 10$$

$$= 240\pi \text{ N/C}$$

$$= 753.6 \text{ V/m}$$

$1\,cal = 4.18\,J$
$= 4.2\,J$

(b) $Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{377}{2} = 120\pi$

$\sigma = 0 \Rightarrow$ Non-conducting

Hence use $Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{4\epsilon_0}}$ $\quad \epsilon_0 = 4\epsilon_0$

A3 $\quad I = \frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{\text{Power}}{\text{Area}}$

$r = 1.5 \times 10^{11}\,m$ $\quad$ Area $= 4\pi r^2 = 4\pi(1.5\times10^{11})^2$

$P = 3.8 \times 10^{26}\,W$

$(W = J/s)$

$\therefore I = \frac{3.8\times10^{26}\,W}{4\pi(1.5\times10^{11})^2} = \frac{3.8\times10^{26}}{28.26\times10^{22}}$

$= 1.34\times10^{3}\,J/\sec\,m^2$

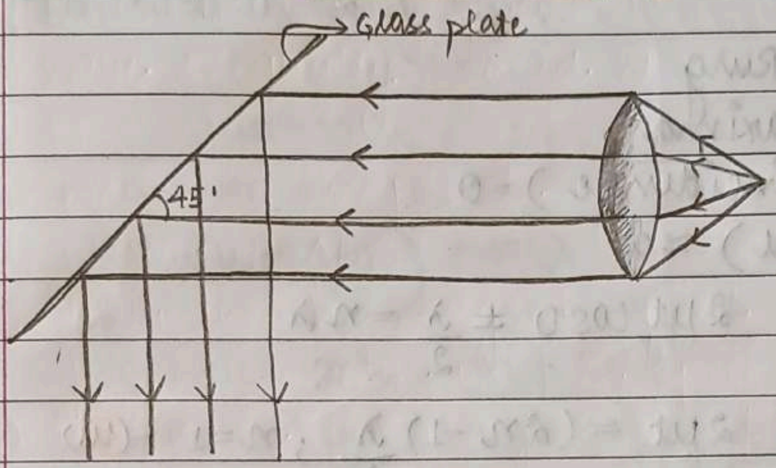
unit changes to $J \rightarrow cal$
$\sec \rightarrow min$
$m^2 \rightarrow cm^2$

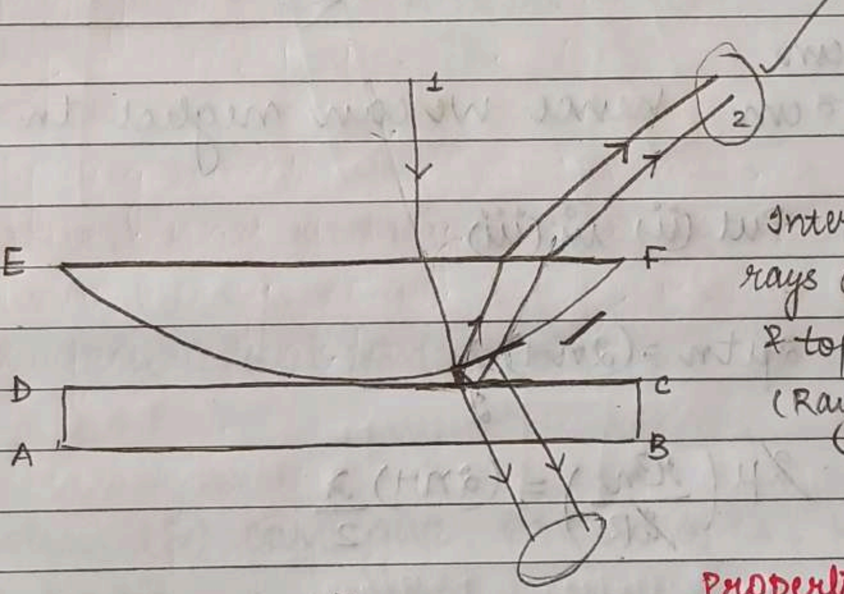
$\frac{1.34\times10^3 \times 60}{4.18 \times \cancel{60} \times 100 \times 100} = 1.914\,cal/cm^2\,min$

Date :- 16 Jan, 2023

Newton Rings

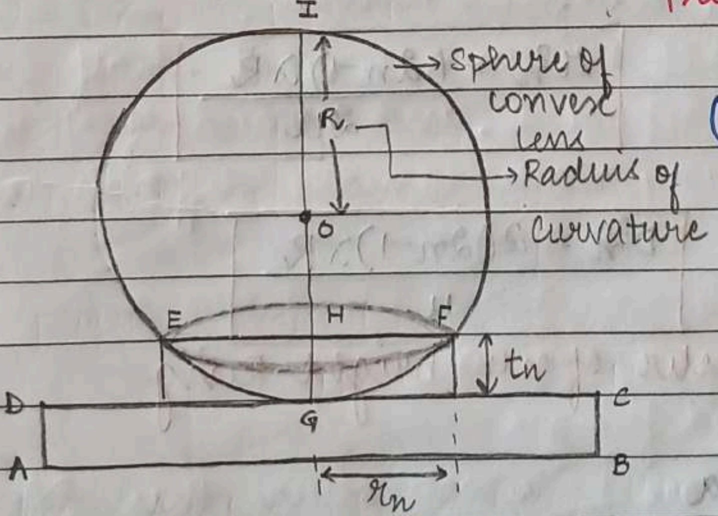
↳ Eg of wedge shaped thin film.





plano convex lens.

Interference is between rays of plano convex & top of AC (Ray 1 & 2)



Property of circle

$(IH)(HG) = EH \times HF$

$(2R - t_n)\, t_n = r_n^2$ — (i)

$2R t_n = r_n^2$

$\boxed{t_n = \dfrac{r_n^2}{2R}}$ (ii)

for wedge shaped film

$2\mu t \cos(r+\theta) \pm \dfrac{\lambda}{2} = P.D$

(only t is variable)

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for max,

$$2\mu t\cos(r+\theta) \pm \frac{\lambda}{2} = n\lambda$$

for Newton Ring

for Maxima

r (normal incidence) $= 0$

θ (very small) ≈ 0

$$2\mu t\cos 0 \pm \frac{\lambda}{2} = n\lambda$$

$$2\mu t = (2n-1)\frac{\lambda}{2} \;;\; n = 1 \quad \text{-(iii)}$$

$\left[\begin{array}{l} R = 100\,cm \\ t = 10^{-3}\,cm \quad \text{hence we can neglect } t_n \text{ in (i)} \end{array} \right]$

Put (ii) in (iii)

$$2\mu t_n = (2n-1)\frac{\lambda}{2}$$

$$\cancel{2}\mu\left(\frac{r_n^2}{\cancel{2}R}\right) = (2n-1)\frac{\lambda}{2}$$

$$\boxed{r_n^2 = \frac{(2n-1)\lambda R}{2\mu}}$$

$$\boxed{D_n^2 = \frac{2(2n-1)\lambda R}{\mu}}$$

Diameter of nth bright Ring

for minima

$$2\mu t\cos(\theta+r) \pm \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos(\theta + r) = n\lambda$$

For newton Ring

$$2\mu t = n\lambda$$

$$2\mu\left(\frac{r_n^2}{2R}\right) = n\lambda$$

$$r_n^2 = \frac{n\lambda R}{\mu}$$

$$\boxed{D_n^2 = \frac{4n\lambda R}{\mu}} \quad -(iv)$$

Diameter of nth dark Ring.

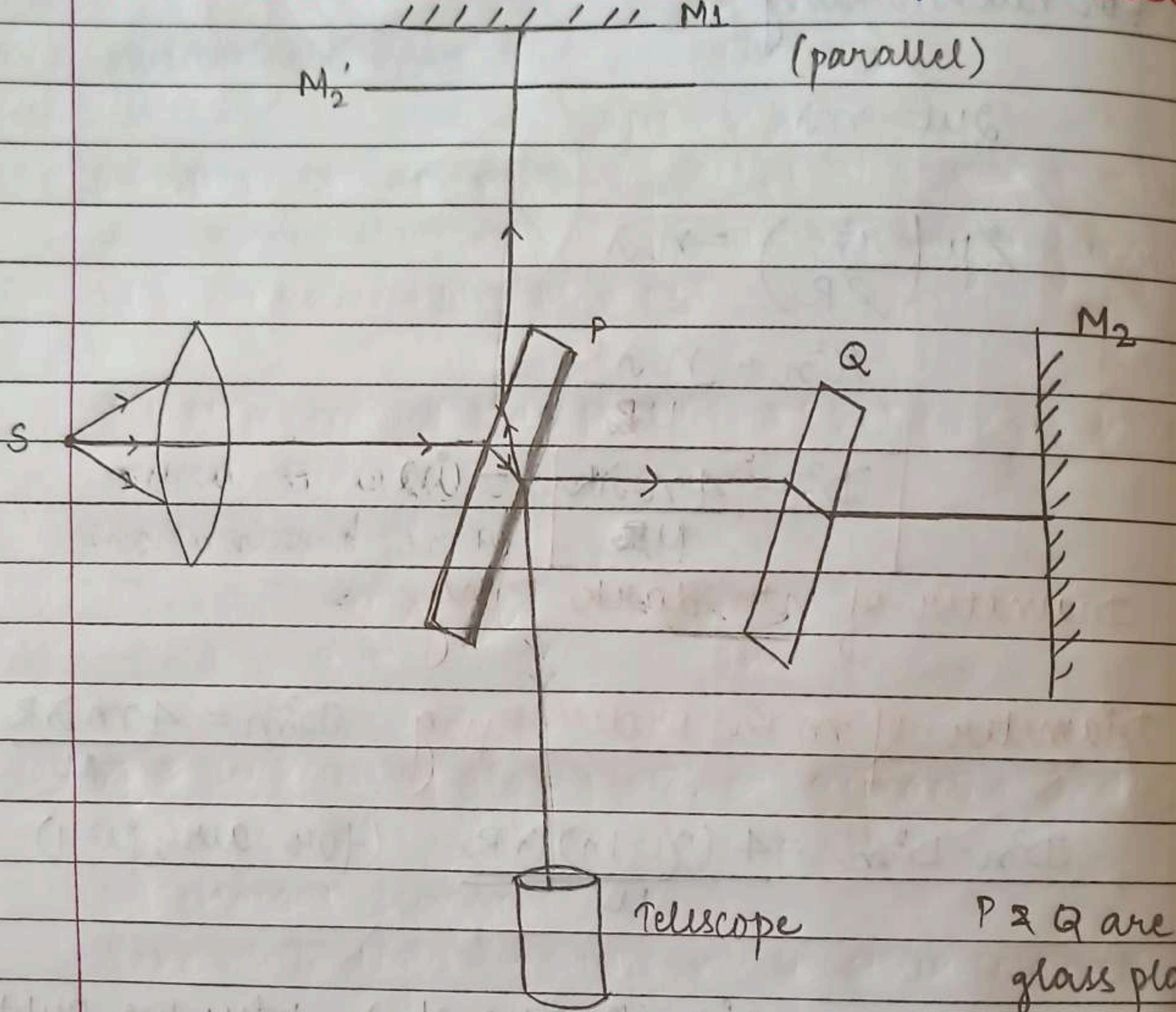
Diameter of mth order Ring $D_m^2 = \frac{4m\lambda R}{\mu}$

$$D_n^2 - D_m^2 = \frac{4(n-m)\lambda R}{\mu} \quad (\text{for air}, \mu = 1)$$

Q. When (iv) can give value of λ, why we subtract D_n^2 & D_m^2

Date : - 15 Jan 2023

Michelson Interoferometer



used to study Interfence pattern.
back surface of P is semi silvered, half Intensity reflect, half refracted, and no need of $\lambda/2$ in calculation
Q is compensatory plate to make diff path in air & glass same. ($t(\mu-1)x$)

Types of fringes

(a) Non - localised fringes

M_1 & M_2 are $\perp$ to each other, then M_1 & M_2' are parallel to each other. (M_2' is image of M_2 through P). Rays of reflected & refracted from back of P shows pattern.

Q. diff b/w fringes of MI & Newton's Ring

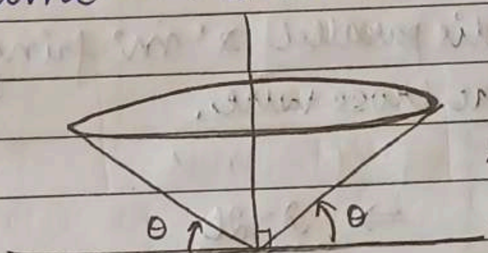
A. Newton's Rings :- same thickness
MI :- same i.

$2\mu t \cos r = n\lambda$ (M_1 & M_2' are parallel thin films)
$= n = 0, 1, 2$ max

$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$ ($n = 1, 2, 3$) min

Shape of fringes :- $2\mu t \cos r$
$\cos r$ is variable here.

r is same = i is same

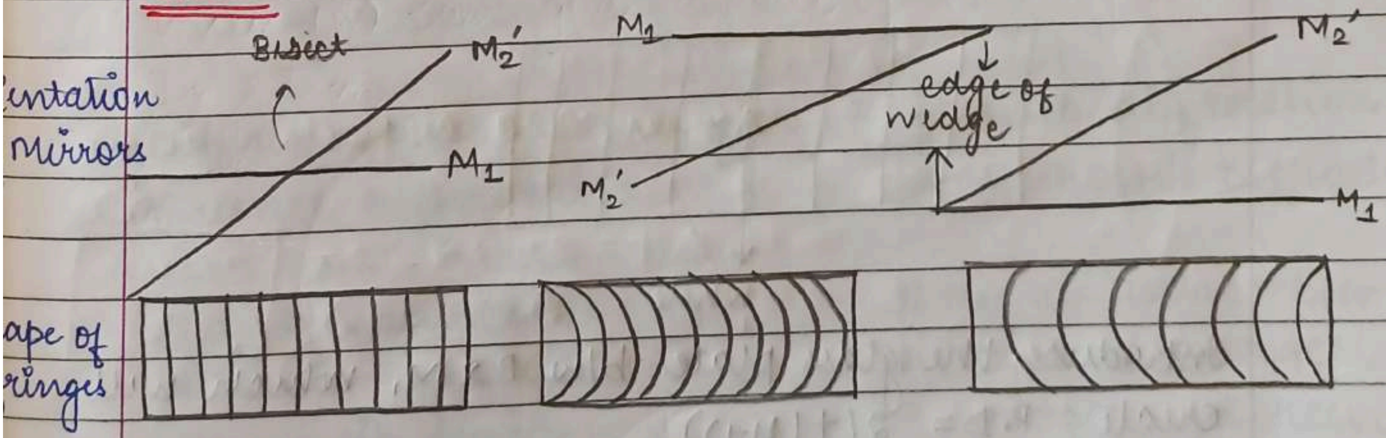


Locus of point having same inclination = circle.

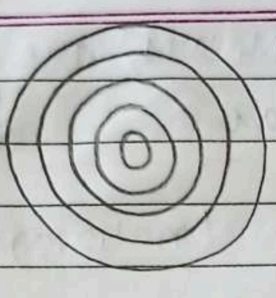
Locus of point having same inclination = Haidenges Fringes.

(b) Localised fringes
$M_1 \not\perp M_2$

Case 1 :- Case 2 :- Case 3 :-



Applications
(a) To find the λ of any monochromatic source
place $M_1 \perp M_2 \Rightarrow$ circular fringes like in Newton's Ring.



for r for central maxima

$r = 0$

$\cos 0 = 1$

$2\mu t = n\lambda$

$\mu = 1 \Rightarrow 2t = n\lambda$ $\left\{ \begin{array}{l} n=1,\ t=\lambda/2; \quad ,\ t = 3\lambda/2,\ t = 5\lambda/2 \\ \text{will make} \quad \text{fringes} \end{array} \right\}$

t is thickness b/w M_1 & M_2'

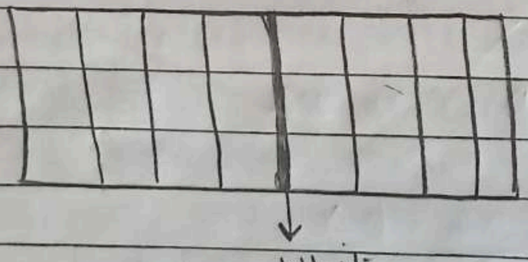
Let M_1 is moved 'l' dis parallel & 'm' fringes have passed through the cross wire.

$$\frac{m\lambda}{2} = l \Rightarrow \lambda = \frac{2l}{m}$$

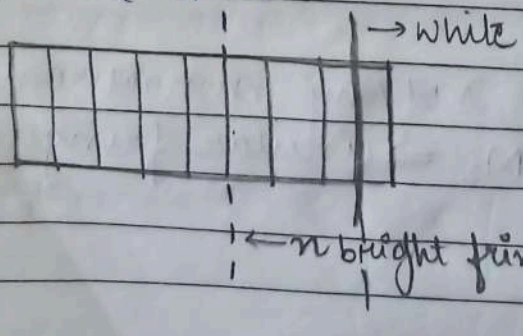
(2) thickness of glass plate. or refractive Index

Get straight fringes

Take white light → centre white other coloured



Introduce the glass plate b/w P & M, which will create P.D = $2(t(\mu-1))$

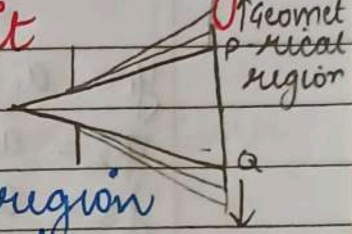


we move $n\lambda = \cdot 2t(\mu-1)$

Date :- 19 Jan 2023

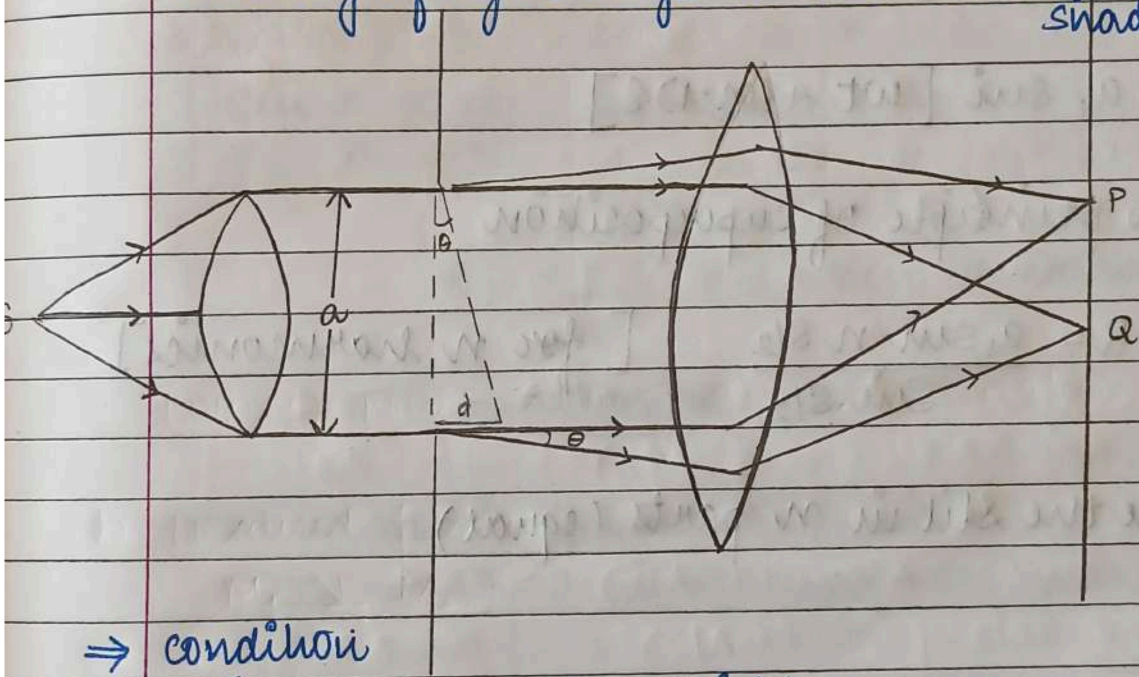
Source
Screen
Obstacle

Differaction due to single slit



FRAUNHOFFER'S DIFFRACTION

Bending of light rays in Geometrical region shadow



$\sin\theta = \frac{d}{a}$

$a\sin\theta = d$

⇒ condition

size of obstacle = λ of light

Difference b/w fraunhoffer & fresnel diffraction

Fresnel Differaction	Fraunhoffer Diffraction
If dis b/w source & obstacle & obstacle & screen is finite	If the distance b/w source and obstacle & obstacle & screen is ∞
No lens is used	use of convex lens.
Spherical wavefronts	(At ∞ dis, rays become ll^{el}) Plane wavefronts

Difference b/w Interference & Diffraction (HW)

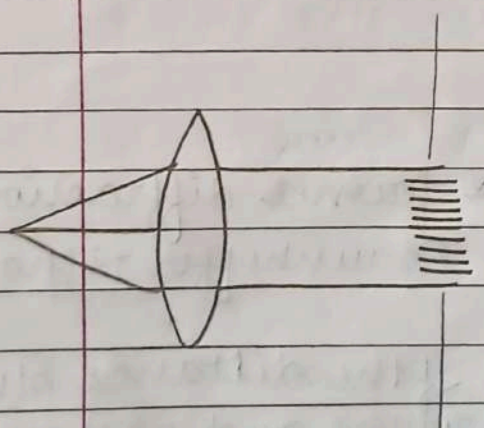
Continuing diagram

$y_1 = a_1 \sin \omega t$
$y_2 = a_1 \sin(\omega t + \delta)$
$y_3 = a_1 \sin(\omega t + 2\delta)$
$\vdots$
$y_n = a_1 \sin[\omega t + (n-1)\delta]$

Acc to principle of Superposition

$$R = \frac{a_1 \sin n\delta/2}{\sin \delta/2} \quad [\text{for } n \text{ harmonics}]$$

Divide the Slit in n parts (equal)



Δx in n equal parts $= \frac{a\sin\theta}{n}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left[\frac{a\sin\theta}{n}\right]$$

Dimension same = amplitude same
& dis b/w source & obstacle is ∞ for all

$$R_1 = \frac{a_1 \sin n\left[\frac{2\pi}{\lambda}\left(\frac{a\sin\theta}{2n}\right)\right]}{\sin\left[\frac{2\pi}{\lambda}\left(\frac{a\sin\theta}{2n}\right)\right]}$$

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$$R_p = \frac{a_1 \sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} \frac{\sin\theta}{n}\right)}$$

$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

$$R_p = \frac{a_1 \sin\alpha}{\sin\alpha/n} = \frac{a_1 \sin\alpha}{\alpha/n} = \frac{n a_1 \sin\alpha}{\alpha} = \frac{A \sin\alpha}{\alpha}$$

$$\left[\sin\frac{\alpha}{n} \approx \frac{\alpha}{n}, \ n \rightarrow \text{Large}, \ A = n a_1\right]$$

$R_p^2 \propto I_p$ (p is when deviated at θ)

$$I_p = \frac{A^2 \sin^2\alpha}{\alpha^2}$$

Now we need $I_{p\,max}$ & $I_{p\,min}$

$$\frac{\partial I_p}{\partial \alpha} = A^2\left[\frac{2\sin\alpha\cos\alpha\,\alpha^2 - \sin^2\alpha\, 2\alpha}{\alpha^4}\right] = 0$$

$$\frac{\partial I_p}{\partial \alpha} = \frac{2A^2 \sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

Double diff for max & min.

for minima $\sin\alpha = 0$ but $\alpha \neq 0$ (bcz $\alpha = 0$ for maxima)

$\sin\alpha = \pm m\pi$; $m = 1, 2, 3, \ldots$

$$\frac{\pi a \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{a\sin\theta = \pm m\lambda}, \ m = 1, 2, 3$$

for maxima : $\alpha\cos\alpha - \sin\alpha = 0$ (secondary maxima)

$$\alpha = \tan\alpha$$

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$y = \alpha$ $\quad$ $y = \tan\alpha$

POI.

$\alpha = 0$ is primary maxima

other values $\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

(secondary max)

Imp

Q Show the ratio of Intensities of Primary & Secondary maxima is given below.

A. $I_p = \lim_{\alpha \to 0} \frac{A^2 \sin^2\alpha}{\alpha^2} = A^2 = I_0$

$I_{S_1} = \lim_{\alpha \to 3\pi/2} \frac{A^2 \sin^2(3\pi/2)}{(3\pi/2)^2} = \frac{I_0 4}{9\pi^2}$

$I_{S_2} = \lim_{\alpha \to 5\pi/2} \frac{A^2 \sin^2(5\pi/2)}{(5\pi/2)^2} = \frac{I_0 4}{25\pi^2}$

$I_{S_3} = \lim_{\alpha \to 7\pi/2} \frac{A^2 \sin^2(7\pi/2)}{(7\pi/2)^2} = \frac{I_0 4}{49\pi^2}$

$I_p : I_{S_1} : I_{S_2} : I_{S_3} = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$

23 Jan 2023

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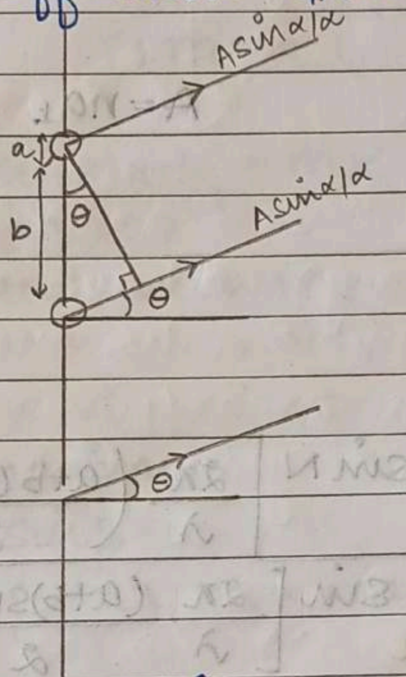
FRANHOFFER DIFFERACTION

due to N-Slits

(Diffration Grating)

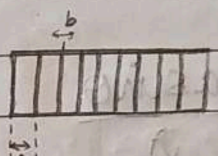


$a+b$ = grating element

$\Delta x = (a+b)\sin\theta$

$\Delta\phi = \frac{2\pi}{\lambda}\left[(a+b)\sin\theta\right]$

A convex lens is used b/w source & obstacle to make the rays parallel and other is used b/w obstacle & screen to converge Rays.



10 lines in 1 cm ; a = width of transparent part
b = width of opaque part

$10(a+b) = 1\,cm \Rightarrow a+b = \frac{1\,cm}{10\text{ lines}}$

If 1 cm contains 100 lines

$a+b = \frac{1\,cm}{100\text{ lines}}$

If 1 inch contains 15000 lines

$a+b = \frac{1\text{ inch}}{15000\text{ lines}}$

n no-of lines = n slits

$$R = a_1 \frac{\sin^n \delta/2}{\sin \delta/2}$$

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} \qquad A = na_1, \; \alpha = \frac{\pi a \sin\theta}{\lambda}$$

For N slits,

$$a_1 = \frac{A\sin\alpha}{\alpha}$$

Hence
$$R_p = \frac{A\sin\alpha}{\alpha} \frac{\sin N\left[\frac{2\pi}{\lambda}\left(\frac{a+b(\sin\theta)}{2}\right)\right]}{\sin\left[\frac{2\pi}{\lambda}\frac{(a+b)\sin\theta}{2}\right]}$$

Intensity due to single slit

$$I_p = \frac{A^2\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta} \rightarrow \text{Impact of N slits}$$

$$\beta = \frac{\pi(a+b)\sin\theta}{\lambda} \qquad \alpha = \frac{\pi a\sin\theta}{\lambda}$$

$$I_p' = \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\frac{\partial I_p'}{\partial \beta} = \frac{2\sin N\beta \cos N\beta \, N \sin^2\beta - \sin^2\beta N \, 2\sin\beta\cos\beta}{\sin^4\beta}$$

$$= \frac{2\sin N\beta}{\sin\beta}\left[\frac{N\cos\beta N\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

for minima

$\sin N\beta = 0$ but $\sin\beta \neq 0$

$N\beta = m\pi$; $m = \pm$

If $\sin\beta = 0 \Rightarrow \frac{\pi(a+b)\sin\theta}{\lambda} = n\pi$

$n = 0, 1, 2$

but $n = 0 \Rightarrow$ maxima

Hence $\sin\beta \neq 0$

$$\frac{N\cancel{\pi}(\sin\theta)\,a+b}{\lambda} = m\cancel{\pi}$$

$$\boxed{(a+b)\sin\theta = \frac{m\lambda}{N}} \text{ —(i)}$$

$m \neq 0, N, 2N, 3N$ *

for maxima

$\sin\beta = 0$

$\beta = n\pi$, $n = \pm 0, \pm 1, \pm 2$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = n\pi$$

$$\boxed{(a+b)\sin\theta = n\lambda} \text{ —(ii)} \quad n = 0, 1, 2, 3$$

* (i) = (ii) for $m = 0, N, 2N, 3N$ hence removed values from minima.

$$I'_p = \frac{\sin^2 N\beta}{\sin\beta}$$

$$R'_p = \lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin\beta} = N$$

$$I'_p = N^2$$

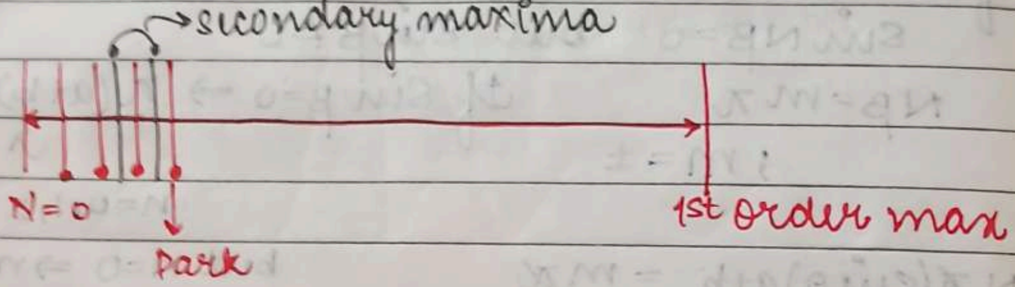
for N slits, intensity is N^2 times from single slit

$m = 0 \rightarrow$ zero order

$m = N \rightarrow 1^{st}$ order max

$m = 2N \rightarrow 2^{nd}$ order max

$1, 2, 3 - - - N-1 \longrightarrow$ dark



between 2 max (primary) we have N-1 dark (minima) & b/w 2 minima lies one secondary maxima. So between 2 max (primary) lies N-1 dark & N-2 secondary maxima

For secondary maxima

$$N\cos\beta N \sin\beta - \sin N\beta \cos\beta = 0$$

$$\boxed{N\tan\beta = \tan N\beta} \longrightarrow \text{cond}^n \text{ for secondary maxima.}$$

Absent Spectrum :-

Maxima :- $(a+b)\sin\theta = n\lambda$, ~~$\theta$~~, $n = 0, 1, 2, 3$

Minima due to single slit $= a\sin\theta = \pm m\lambda$

If both condition holds true for some m ~~or~~ or n simultaneously then that order maxima would be absent.

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n}{m} \Rightarrow \frac{a+b}{a} = \frac{n}{m}$$

$$n = m\left[\frac{a+b}{2}\right]$$

If $a = b$; $n = 2m \Rightarrow$ Even order spectrum

$n = 2, 4, \ldots$

Dispersive power :- How θ varies with λ is D.P

$(a+b)\sin\theta = n\lambda$ $\theta = \angle$ of diffraction

$(a+b)\cos\theta\, d\theta = n\, d\lambda$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

θ is small , $d\theta \propto d\lambda$

↳ const $\frac{d\theta}{d\lambda}$

$d\theta \propto d\lambda \Rightarrow$ Normal spectrum.

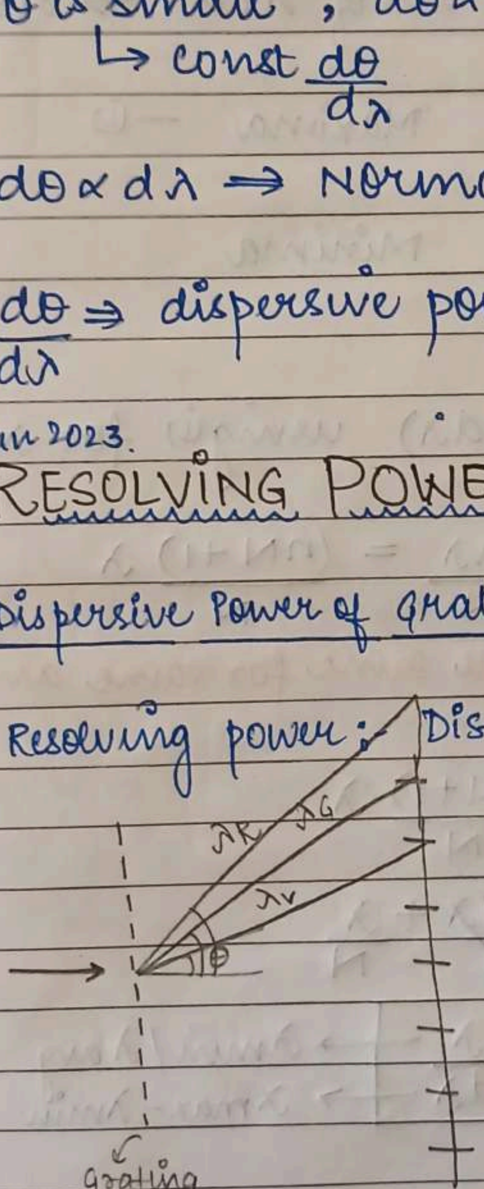
$\frac{d\theta}{d\lambda} \Rightarrow$ dispersive power.

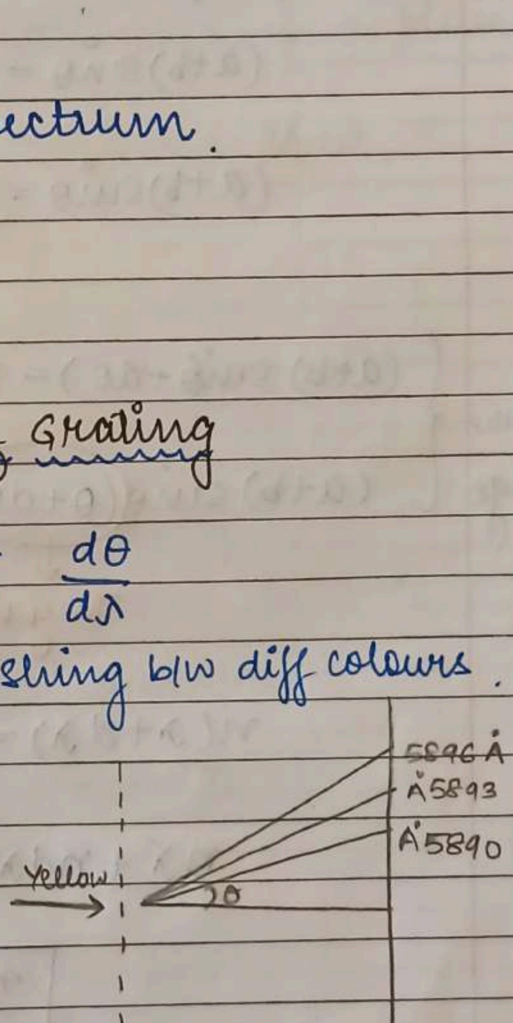
Date :- 25 Jan 2023.

RESOLVING POWER of Grating

Dispersive Power of grating :- $\frac{d\theta}{d\lambda}$

Resolving power :- Distinguishing b/w diff colours.

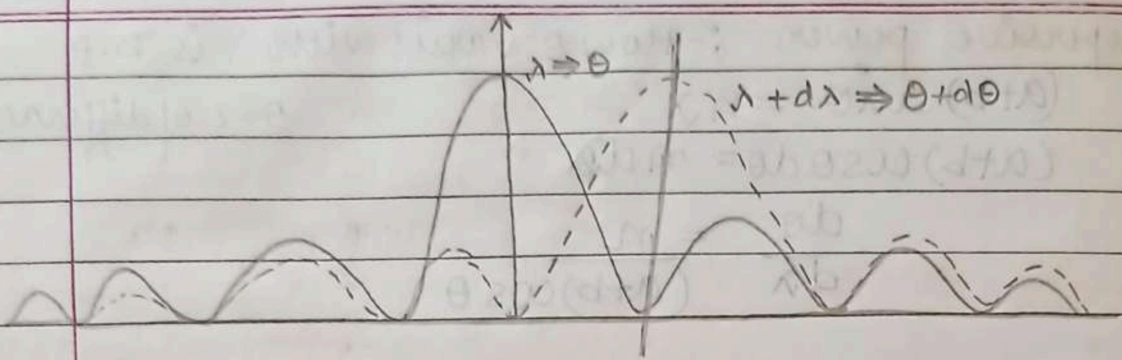




Yellow has 3 λ 5890, 5896 Å
5893 Å

$d\theta \downarrow$

* Rayleigh criteria for Resolution
The minimum condition is when minima of λ falls on maxima of $\lambda + d\lambda$.



Minima of λ falls on Max of $\lambda + d\lambda \Rightarrow$ resolveable

$$(a+b)\sin\theta = n\lambda \quad \text{Maxima} \quad -(i)$$

$$(a+b)\sin\theta = \frac{m\lambda}{N} \quad \text{Minima}$$

for nth fringe
$$\begin{cases} (a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda) & \text{using (i) for } \lambda+d\lambda \\ (a+b)\sin(\theta+d\theta) = \dfrac{m\lambda}{N} = \dfrac{(nN+1)\lambda}{N} \end{cases}$$

They will fall same for same angle

$$n(\lambda+d\lambda) = \frac{(nN+1)}{N}\lambda$$

$$\cancel{n\lambda} + n\,d\lambda = \frac{n\cancel{N}\lambda}{\cancel{N}} + \frac{\lambda}{N}$$

$$\boxed{nN = \frac{\lambda}{d\lambda}}$$

$\lambda \to \lambda_{min} / \lambda_{avg}$

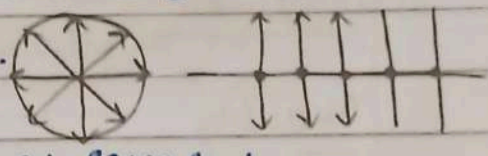
$d\lambda \to \lambda_{max} - \lambda_{min}$

N = no. of lines | slits

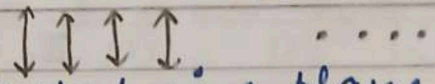
n = nth order

POLARISATION :-

(a) Unpolarised light :- $\vec{E}$ field can be oriented at any $\angle$ wrt propagation vector.



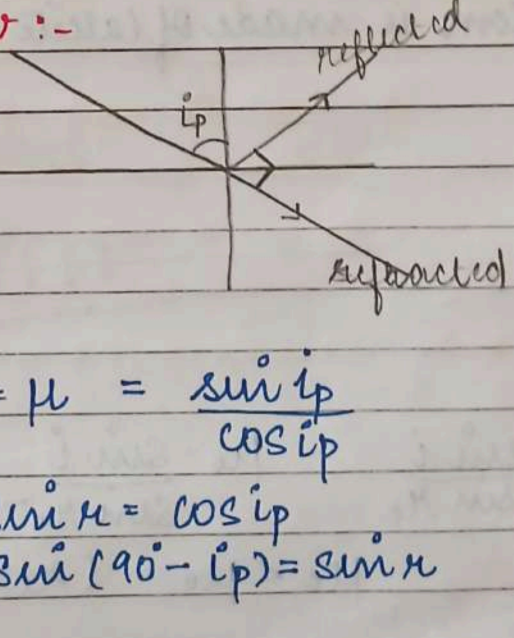
(b) Polarised light :- $\vec{E}$ is alligned at same $\angle$.



(c) Plane Polarised light :- $\vec{E}$ oriented in a plane

Methods to produce Plane Polarised light.

Brewster's law :-



$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$$

Snell's law

$$\frac{\sin i_p}{\sin r} = \mu = \frac{\sin i_p}{\cos i_p}$$

$$\sin r = \cos i_p$$

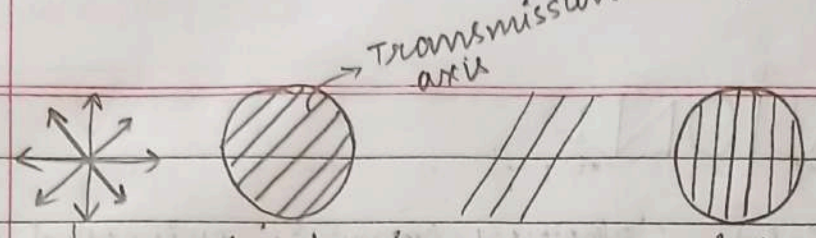
$$\sin(90^\circ - i_p) = \sin r$$

$$\boxed{90 = i_p + r}$$

Hence $\angle$ b/w reflected & refracted ray is 90°

** Q. How you will distinguish b/w ppl, partially pl, eliptical, circular, unpolarised.

MALUS LAW



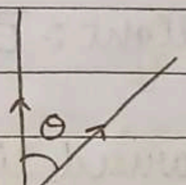
Intensity

I_1 I_2

$I_2 = \frac{I_1}{2}$

$I_p = \frac{I_{unc}}{2}$

$\boxed{\text{Derivation}}$



$I_3 = I_2 \cos^2\theta$

$I_A = I_p \cos^2\theta$

$\theta = 0°, 180° \Rightarrow I_A = I_p$

$\theta = 90°, 270° \Rightarrow I_A = 0$

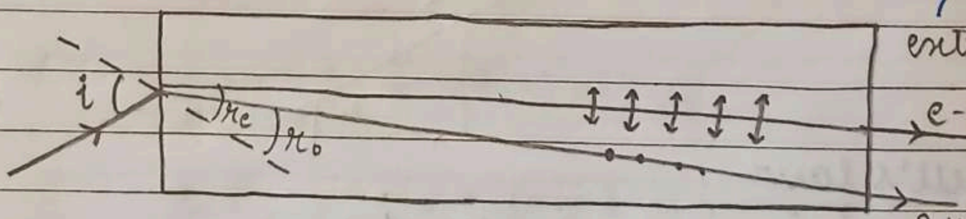
DOUBLE REFRACTION

2 refracting light in certain crystals like borax, topaz, Calcite, Quartz.

Nicole prism is made of Calcite

$\boxed{l = 3b}$

doesnot follow snells.



ordinary ray

$\mu_e = \frac{\sin i}{\sin r_e}$ $\mu_o = \frac{\sin i}{\sin r_o}$ → follows snells law

$\mu_e < \mu_o$ $(r_e > r_o)$

TIR

$i > \theta_c$ & Denser → Rarer.

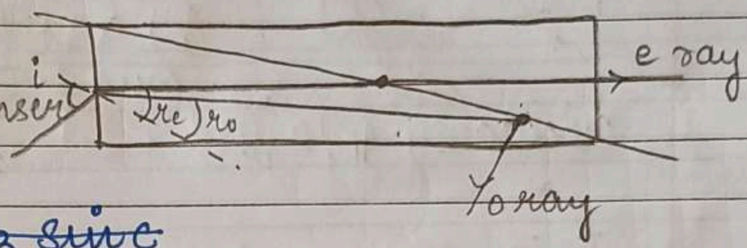
we choose a medium having μ b/w μ_e & μ_o which is canada balsam

2 corners → 3 ∠s of 102°
(diagonally opposite) → blunt corner
rest 1 → 102°, 2 → 78°

Calcite :- crystallised $CaCO_3$.
Rhombohedral
(6 faces, all ∥gm, 102°, 78°)

CLASSTIME Pg. No.

$\mu_o = 1.65$) rarer
$\mu_{CB} = 1.55$
$\mu_e = 1.48$) ~~ord~~ denser



~~$\mu_o \sin 90° = \mu_{CB} \sin c$~~

$[\mu_o \sin c = \mu_{CB} \sin 90°]$ $\quad \mu = \frac{1}{\sin c} \Rightarrow \sin c = \frac{1}{\mu_o/\mu_{CB}} = \frac{\mu_{CB}}{\mu_o} = \frac{1.55}{1.65}$

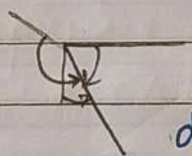
≈ 69

Terms :-

(a) Blunt corner
where the 3 ∠s meeting at a corner all are obtuse.

(b) Optic axis
an axis through blunt corner & equally inclined to 3 faces/∠s.

(c) Principal section
A plane containing optic axis & ⊥ to opposite phases.



(d) Anisotropic crystal — different μ in different directions. (non-uniform) particle arrangement.

- Uniaxial
  One optic axis
  calcite, quartz
- Biaxial
  2 optic axis
  Mica, borax, topaz
  (both e ray)

Isotropic → same n in diff dir.

Uniaxial

- Positive crystal
  Quartz
  $r_e < r_o$
- Negative crystall
  Calcite.
  $r_e > r_o$

$$\boxed{r \propto \frac{1}{\mu} \propto v}$$

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~~Pol~~ Uniaxial

QUARTER and HALF WAVE PLATE

↳ rotate plane of polarisation

Circularly polarised $\xleftrightarrow{\text{Quarter wave plate}}$ Elleptically polarised

(i) Ve ray & Vo ray in crystal ⇒ different
after refraction ⇒ same

(ii) $\vec{E} \parallel$ optical axis ⇒ e ray
$\vec{E} \perp$ optical axis ⇒ o ray

(iii) If a plane is cut ∥ to optic axis and
~~(iii)~~ If light is incident ⊥ to optic axis then e ray & o ray will travel along same dir but with diff speeds & plane of polarisation.

Now when they emerge back after refraction, there will be a Δx & $\Delta \phi$ b/w them

If $\Delta x = \frac{\lambda}{4}$ ⇒ Quarter wave plate

$\Delta x = \frac{\lambda}{2}$ ⇒ Half wave plate.

Negative crystal

$\mu_o > \mu_e$

If $\mu_o t - \mu_e t = \lambda/4$
Quarter wave plate

If $\mu_o t - \mu_e t = \frac{\lambda}{2}$

half wave plate

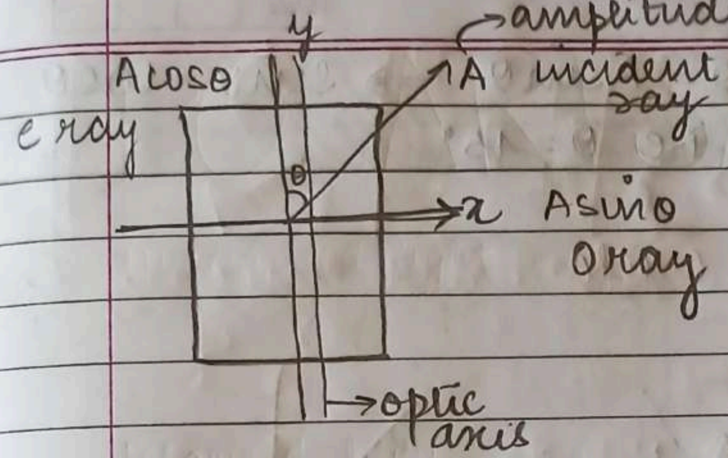
Positive crystal

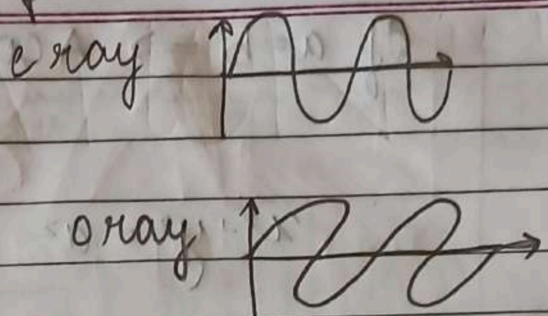
$\mu_e > \mu_o$

$\mu_e t - \mu_o t = \lambda/4$
Quarter wave plate

$\mu_e t - \mu_o t = \frac{\lambda}{2}$

half wave plate





δ = phrase diff

$$y = \underbrace{A\cos\theta}_{b} \sin\omega t \qquad x = \underbrace{A\sin\theta}_{a} \sin(\omega t + \delta)$$

$$\frac{y}{b} = \sin\omega t \qquad \frac{x}{a} = \sin(\omega t + \delta)$$
$$= \sin\omega t \cos\delta + \cos\omega t \sin\delta$$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\,\sin\delta$$

$$\left[\frac{x}{a} - \frac{y}{b}\cos\delta\right]^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\delta}{ab} = \sin^2\delta} \rightarrow \text{ellipse eqn}$$

(i) $\delta = 0$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} = \frac{y}{b} \Rightarrow \boxed{y = \frac{b}{a}x}$$

(ii) $\delta = \pi/2$ $\qquad \lambda/4 = \Delta x$ $\qquad$ Quarter wave plate

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \text{ellipse}$$

funct^n of Quarter wave plate

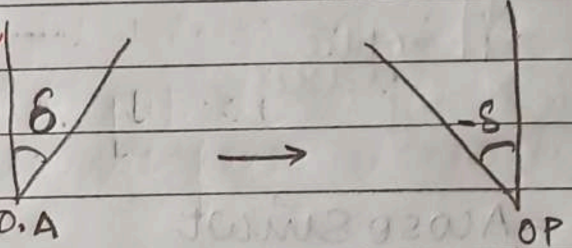
If $a=b \Rightarrow A\cos\theta = A\sin\theta \Rightarrow \sin\theta = \cos\theta$

i.e $\theta = 45°$

$x^2+y^2 = a^2$ circle

(iii) $\delta = \pi$ Half wave plate

$y = \frac{-bx}{a}$



If $\delta = \pi$

before entering $\xrightarrow{-2\delta \text{ change}}$ after emitting

right → dextrorotatory → clockwise

left → laevorotatory → anticlockwise

eg:- Glucose.

extent to polarisation → Polarimeter. (θ)

1) optical activity :- Plane of polarisation Rotation of plane Polarised light. Eg:- Sugar solⁿ

(2) specific rotation $\langle S \rangle_{\lambda}^{Temp(T)} = \frac{\theta}{l\,C}$ θ → rotation produced, C → concentration, l ↓ length of solution tube

(const λ & T)

$\left.\begin{array}{l}\theta \rightarrow {}^{\circ} \\ l \rightarrow dm \\ c \rightarrow g/cc\end{array}\right\}$ units

(3) optical active solution :- which rotates plane of PPL.

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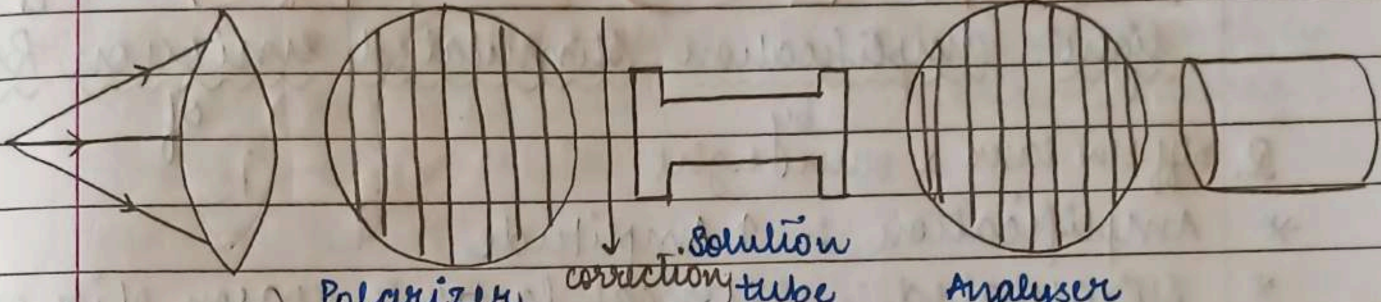
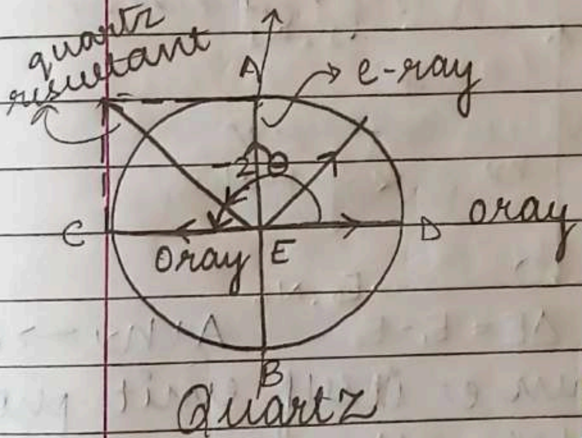
LAURANT'S HALF SHADE POLARIMETER

Normal Polarimeter

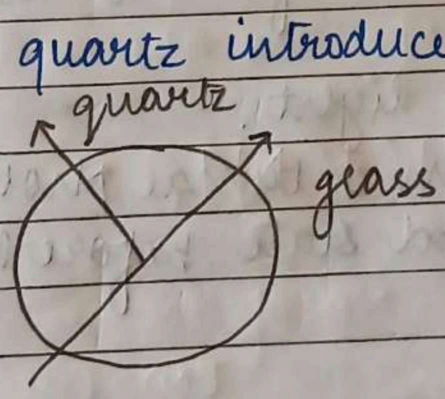


1st pass w/o solⁿ then w solⁿ. rotation in analyser is θ in PPL. Problem :- finding point of maxima



half shade plate
half glass half quartz
funcⁿ :- -2θ rotation
~~uniform~~

optic axis cut ∥ to face.
(refer (ii) & (iii))
thickness such that glass & quartz introduce a $\Delta x = \frac{\lambda}{2}$ / $\Delta\phi = \pi$



$N_1 = N_0 e^{\Delta E / K_B T}$ (Temp)

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photon

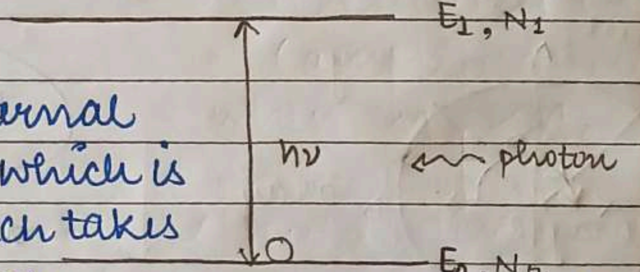
LASER

light amplification by stimulated emission of Radiation

Q. Diff b/w laser & other light

* Amplification :- ↑ Amplitude
* Stimulated :- external factor is reason of its cause. (induced / forced)
* Spontaneous :- (apne aap)

(a) Stimulated absorption :- external energy provided which is gained by e- which takes it to excited state



$\Delta E = E_1 - E_0$ $\quad A + h\nu \rightarrow A^*$

(b) Spontaneous emission :- when e- itself emit photon and comes to ground state after 10^{-8} sec.

$A^* \longrightarrow A + h\nu$

It is observed in orginary light.

(c) Stimulated emission :- when external photon forces e- to come to ground state before life time of atom (10^{-8} sec).

PHOTON ⟶ ABSORPTION (when collide w Ground state)

↓

STIMULATED EMISSION (when collided w. excited state e-)

$A^* + h\nu \longrightarrow A + 2h\nu$ 2 times.

LASER	ORDINARY LIGHTSOURCE
a) can travel in straight line in long distance. Directionality: Same Intensity throughout the distance b/z of high Energy $\Rightarrow \lambda\downarrow \neq$ dis b/w Obstacle for differaction/divergence	a) Intensity $\propto \frac{1}{Dis^2}$ cannot travel in long distance.
b) High Intensity which is attained at 10^{30} K heat.	
c) Monochromatic (one wavelength)	
d) Coherent in nature.	d) Div by Amplitude or wavefront to make it coherent.

Q. Prove mostly Hydrogen in a room is in Ground state.

For hydrogen:- $\frac{-13.6}{n^2}$ eV

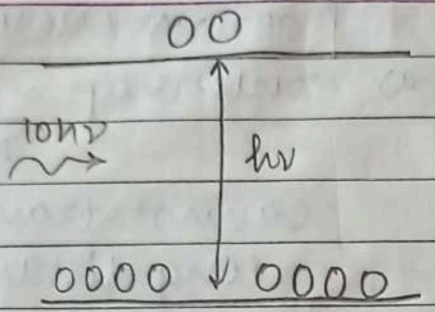
$$E_1 = -13.6\,eV \quad E_2 = -3.4\,eV$$

$$\Delta E = E_2 - E_1 = 10.2\,eV$$

$$K_B T\ (T = 300\,K) = 0.025\,eV$$

$$N_2 = N_1 e^{-10.2\,eV/K_BT}$$

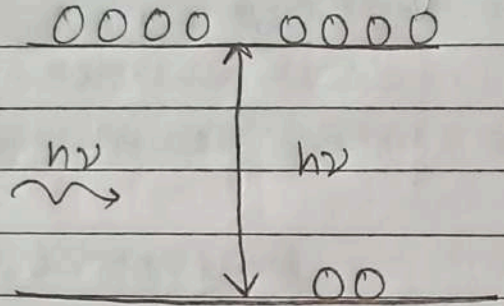
$$N_2 = \frac{N_1}{e^{10.2/0.25}} \approx \frac{N}{e^{400}} \approx 0$$



$\rightsquigarrow$ 4hν

$\Rightarrow$ attenuation (absorbed)

when $n_g > n_e$



$\rightsquigarrow$ 16hν

$\Rightarrow$ Stimulated emission (amplification)

when $n_g < n_e$.

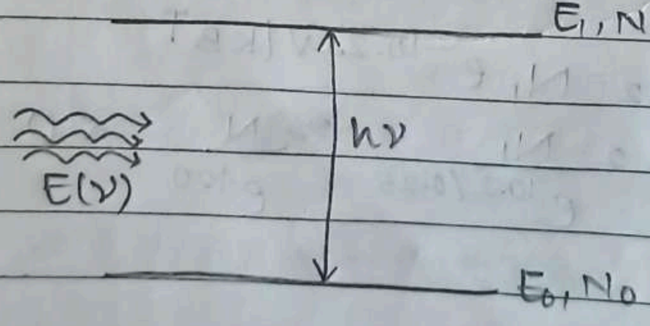
Stimulated emission should dominate on stimulated absorption.

(d) Population Inversion :- when no. of e^- in excited state is more than no. of e^- in ground state.

EINSTEIN COEFFICIENTS :-

No. of upward transition = No. of downwards transition at equilibrium

$E(\nu)$ = Energy density $= \dfrac{\text{No. of photons}}{\text{Area} \times \text{time}} = \dfrac{\text{No. of photons}}{\text{volume}}$ = Intensity



No. of upward transition in Δt time $\propto$ No. of photons = $E(\nu)$

$\propto \Delta t$

$\propto N_0$

No. of upward transition per unit time $= B_{12}\, E(\nu) N_0$

B_{12} → Einstein coeff. of stimulated absorption.

No. of downward transition = spontaneous emission + stimulated emission

spontaneous emission $\propto N_1$

$= A_{21} N_1$

A_{21} → Einstein coeff of spontaneous emission

stimulated emission $\propto N_1$

$\propto E(\nu)$

$= B_{21} N_1 E(\nu)$

B_{21} → Einstein coeff of stimulated emission

At equilibrium.

$$B_{12} E(\nu) N_0 = A_{21} N_1 + B_{21} N_1 E(\nu)$$

$$E(\nu) = \frac{A_{21} N_1}{B_{12} N_0 - B_{21} N_1} = \frac{A_{21}}{B_{12}\left(\frac{N_0}{N_1}\right) - B_{21}}$$

$$= \frac{A_{21} / B_{12}}{\frac{\cancel{B_{12}}}{\cancel{B_{12}}}\left(\frac{N_0}{N_1}\right) - \frac{B_{21}}{B_{12}}} = \frac{A_{21} / B_{12}}{\frac{N_0}{N_1} - \frac{B_{21}}{B_{12}}}$$

$$= \frac{A_{21} / B_{12}}{e^{h\nu / K_B T} - B_{21} / B_{12}} \quad \text{—(1)}$$

(from $N_1 = N_0 e^{-h\nu / K_B T}$) Boltzman law.

Metastable State

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Plank's law of Radiation

$$E(\nu) = \frac{8\pi h\nu^3}{c^3\left[e^{h\nu/K_BT} - 1\right]} \quad -(2)$$

compare (2) & (1)

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3} \quad \& \quad \frac{B_{21}}{B_{12}} = 1$$

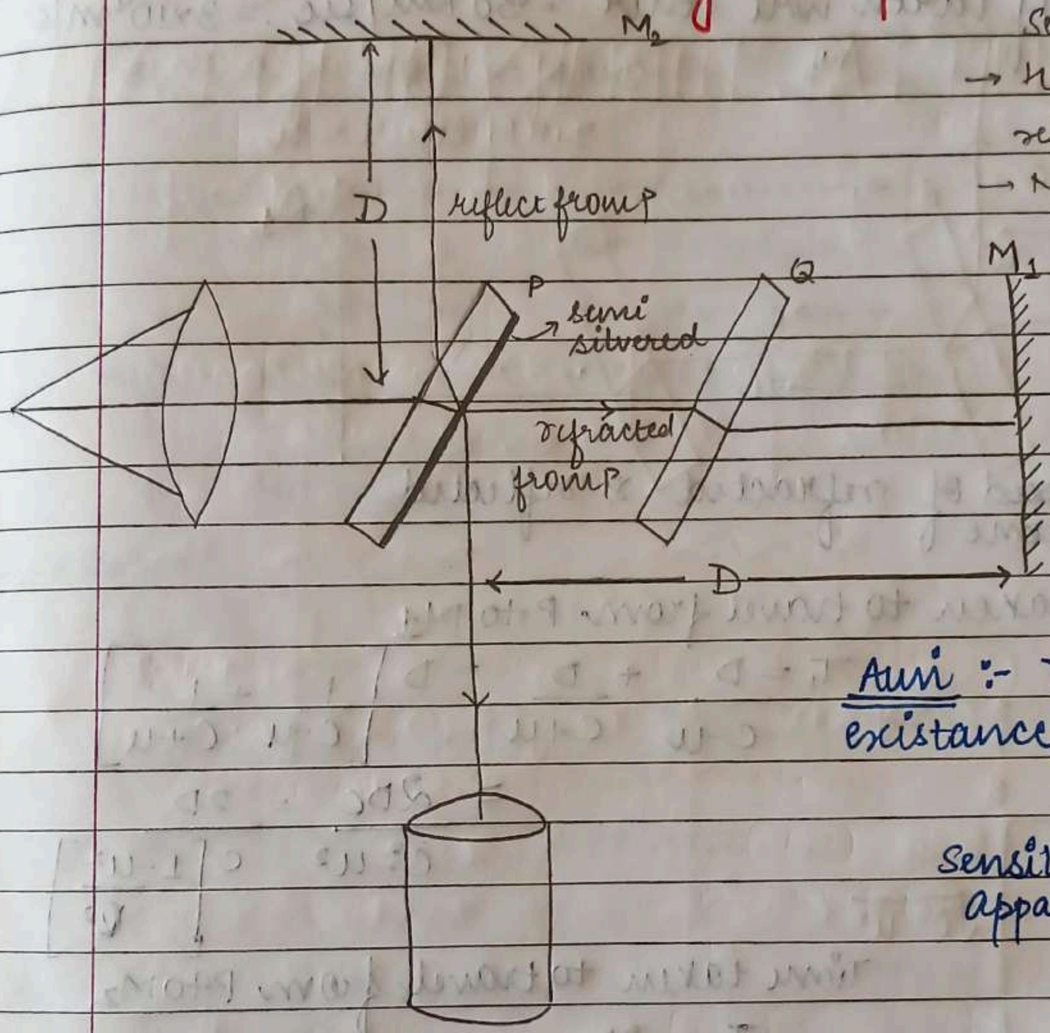
$$\Rightarrow B_{21} = B_{12}$$

*Ques in exam

Date:- 6 Feb 2023

** Imp for paper :- Pkka aayega

Michelson Morley Experiment



Semi silvered
→ half Intensity refracted & reflected
→ No π phase diff

Q
→ To make Δx of refracted & reflected same

Aim :- To prove the existance of ether.

Sensitive apparatus (upto $\frac{1}{100}$)

(1) Inertial reference frame :- Non Rotating, non-accelerating (rest or const v), Newton laws valid

(2) Non-inertial frame of reference :- accelerating or rotating

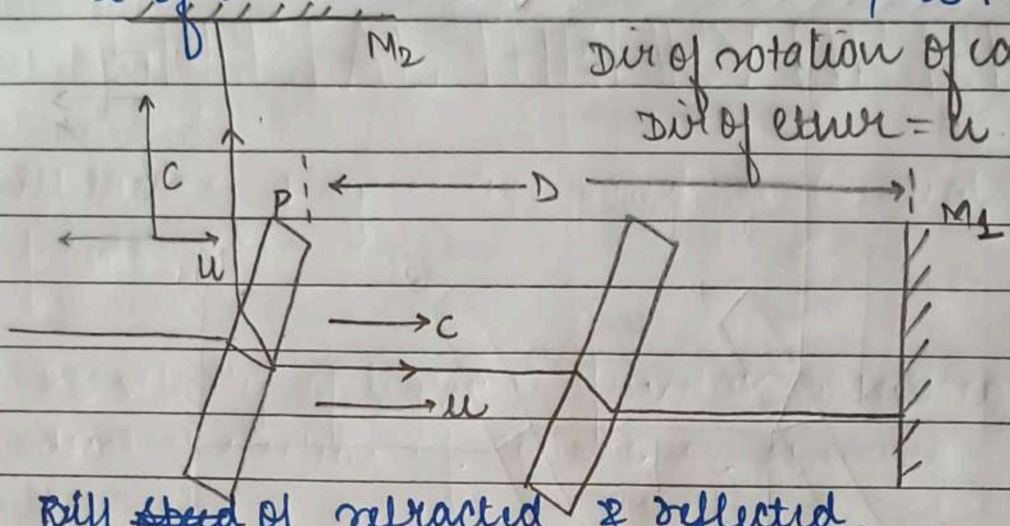
Earth :- acc is less so It is initial reference frame
$a = \omega^2 r \Rightarrow a \approx 3.4$ cm/sec (prove)

→ Light doesnot need medium but when it wasn't discovered, It was considered that a medium (ether) is present b/w Sun & earth for sun rays to travel in space.

Hence 'c' is the velocity of light wrt ether $= 3\times10^8$ m/s

'u' of earth wrt ether $= 30$ km/sec. $= 3\times10^4$ m/s

Dir of rotation of earth = Dir of ether = u

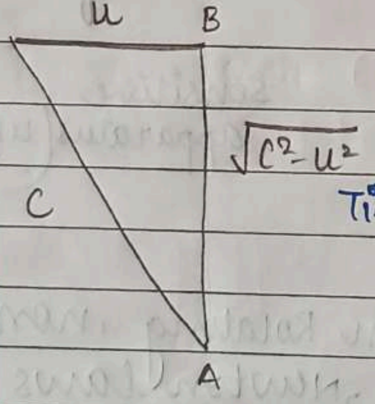


Diff ~~speed~~ time of refracted & reflected.

Time taken to travel from P to M_1

$$T_1 = \frac{D}{c-u} + \frac{D}{c+u} = D\left[\frac{1}{c-u} + \frac{1}{c+u}\right]$$

$$= \frac{2Dc}{c^2-u^2} = \frac{2D}{c\left[1-\frac{u^2}{c^2}\right]}$$



Time taken to travel from P to M_2

$$T_2 = \frac{2D}{\sqrt{c^2-u^2}}$$

$$T_2 = \frac{2D}{c\left[1-\frac{u^2}{c^2}\right]^{1/2}}$$

$$\left(1-\frac{u^2}{c^2}\right)^{1/2} > 1-\frac{u^2}{c^2}$$

$$T_1 > T_2$$

$$\Delta T = \frac{2D}{c}\left[\left(1-\frac{u^2}{c^2}\right)^{-1} - \left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2D}{c}\left[\left(1+\frac{u^2}{c^2}\right) - \left(1+\frac{u^2}{2c^2}\right)\right] = \frac{2D}{c}\left[\frac{u^2}{2c^2}\right] = \frac{Du^2}{c^3}$$

Path diff :- $\frac{Du^2}{c^3} \times c = \frac{Du^2}{c^2}$

He then rotated setup by 90° to find fringe shift

$P.D = \frac{2Du^2}{c^2}$ $\quad D = 11m$

$u = 3\times10^8\, m/s$

$c = 3\times10^8\, m/s$

$n\lambda = \frac{2Du^2}{c^2}$ $\quad \lambda = 6000\,Å$ (taken)

$n \approx 0.4$

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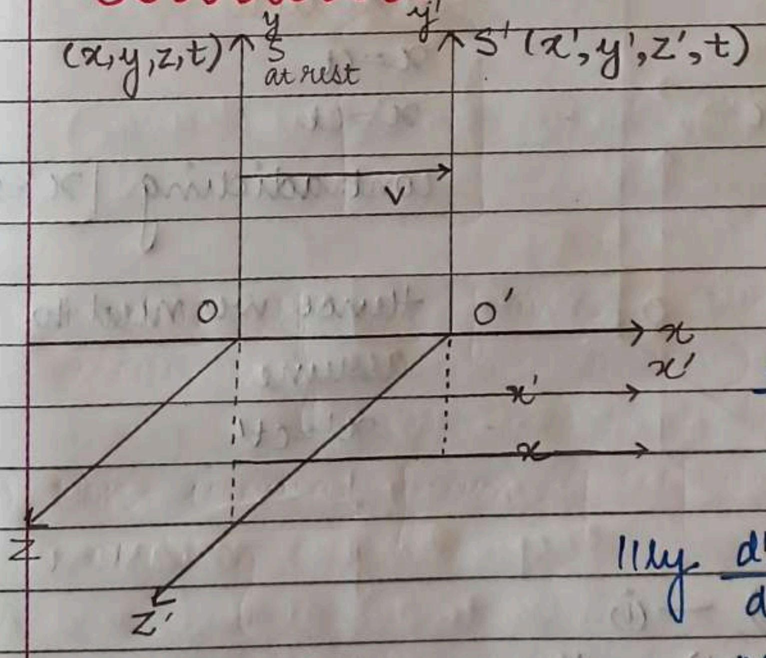
GALILEAN AND LORENTZ EINSTEIN TRANSFORMATION



$x' = x - vt$
$y' = y$
$z' = z$
} Galilean transformation.

$\frac{dx'}{dt} = \frac{dx}{dt} - v$

$u'_x = u_x - v$

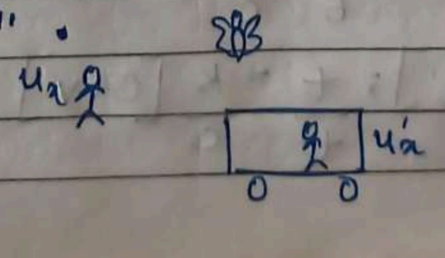
llly $\frac{dy'}{dt} = \frac{dy}{dt} \Rightarrow u'_y = u_y$

llly $u'_z = u_z$

u_x = velocity of object ~~in~~ when observer is in S frame in x direction

u'_x = " " " " " " " S'

" .



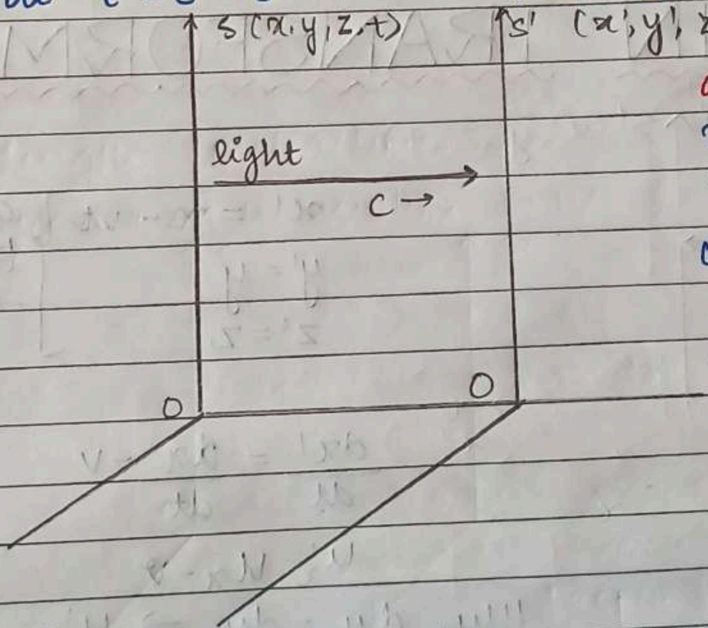
POSTULATES OF

SPECIAL THEORY OF RELATIVITY

(a) All laws of physics are valid in inertial reference frame.

(b) The velocity of light is fundamental constant independent of motion of source or observer.

at $t = t' = 0$; $O = O'$ at that point, flash a light in the x direction



$x = ct$

$x' = ct$

contradicting $\boxed{x' = x - vt}$

Hence we need to assume

$x' = ct'$

$x' = k(x - vt)$ — (i)

$x = k(x' + vt')$ — (ii)

$x = ct$; $x' = ct'$

$ct' = k[ct - vt]$, $\frac{t'}{t} = \frac{k}{c}[c - v]$ — (iii)

$ct = k[ct' + vt']$; $\frac{t}{t'} = \frac{k}{c}[c + v]$ — (iv)

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multiplying ⑧ & ④

$$\frac{t'}{t} \times \frac{t}{t'} = \left[\frac{k}{c}\right]^2 [c^2 - v^2]$$

$$\boxed{\frac{1-k^2}{k^2} = \frac{-v^2}{c^2}}$$

$$1 = \frac{k^2}{c^2}[c^2 - v^2]$$

$$\boxed{k^2 = \frac{c^2}{c^2 - v^2}} \Rightarrow k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 1 - k^2 = 1 - \frac{c^2}{c^2 - v^2} = \frac{-v^2}{c^2 - v^2}$$

Substituting ① & ②

$$x = k[k(x - vt) + vt']$$

$$x = k^2x - k^2vt + kvt'$$

$$kvt' = x[1 - k^2] + k^2vt$$

$$t' = kt + \frac{x(1-k^2)}{kv}$$

$$= k\left[t + \frac{(1-k^2)x}{k^2v}\right] \quad \left[\frac{1-k^2}{k^2} = \frac{-v^2}{c^2}\right]$$

$$t' = k\left[t - \frac{vx}{c^2}\right]$$

Lorentz Transformation

$x' = k(x - vt)$ — (A)

$y' = y$

$z' = z$

$t' = k\left[t - \frac{vx}{c^2}\right] \rightarrow$ (C)

$v \rightarrow -v$

Inverse Lorentz Transf

$x = k(x' + vt')$

$y = y'$

$z = z'$

$t = k\left[t' + \frac{vx'}{c^2}\right]$

$\rightarrow$ (B)

Velocity transformation

$$\frac{dx'}{dt} = k\left[\frac{dx}{dt} - v\right] \rightarrow ⑤ \quad \text{from (A)}$$

$$\frac{dt}{dt'} = k\left[\frac{1+\frac{vU_x'}{c^2}}{}\right] \rightarrow (6)$$

from (B)

$$\frac{dt'}{dt} = k\left[1-\frac{vU_x}{c^2}\right] \rightarrow (7)$$

from (C)

~~Multiply (5) x (6)~~

$$\cancel{\frac{dx'}{dt'} = \frac{dx/dt - v}{1+\frac{vU_x}{c^2}}}$$

Divide 5 ÷ 7

$$\frac{dx'}{dt} \Big/ \frac{dt'}{dt} = \frac{U_x - v}{1-\frac{vU_x}{c^2}}$$

$$U_x' = \frac{U_x - v}{1-\frac{vU_x}{c^2}} \Rightarrow U_x' = \frac{dx'}{dt'}$$

$$U_x = \frac{dx}{dt}$$

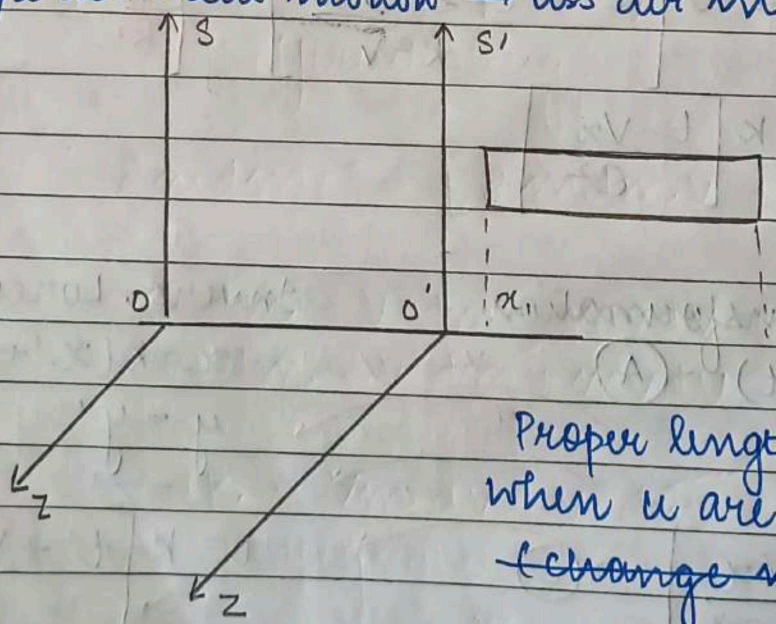
$$U_x' = \frac{U_x - v}{1-\frac{vU_x}{c^2}}$$

put $U_x' = c$

get $U_x = c$

Length contraction

(jis dir mein motion → uss dir mein change)



Proper length: measuring length when u are at rest wrt object ~~(change mon~~

(jiske respect mein obj move krta hai usko ek saath initial & final point measure krna padega)

S' → Rest Rod

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$x_1' = k[x_1 - vt_1]$ $x_2' = k[x_2 - vt_2]$

Proper length $= L_0 = x_2' - x_1'$

$t_1 = t_2$

$x_2' - x_1' = k[x_2 - x_1]$

$L_0 = kL$

$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$ $L < L_0$

Proper Length = Max

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Time Dialation
↳ Inc

Proper time :- (starting and ending on same coordinates) The time interval measured b/w happening of 2 events when you are in a frame where coordinates remains same is proper time. ($\Delta\tau$)

Let $S \rightarrow$ proper time $\rightarrow \Delta\tau \rightarrow t_2, t_1$
$\downarrow x_2 \quad \downarrow x_1$ (coordinates)

$S' \rightarrow ? \rightarrow \Delta\tau'$

$$t_2' = k\left[t_2 - \frac{vx_2}{c^2}\right]$$

$$t_1' = k\left[t_1 - \frac{vx_1}{c^2}\right]$$

$$t_2' - t_1' = k\left[t_2 - t_1\right]$$ (by def^n $x_2 = x_1$)

$$\boxed{\Delta\tau' = k\,\Delta\tau = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}}$$ → proper time

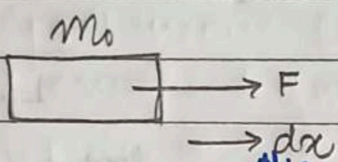
$\Delta\tau < \Delta\tau'$

$\Delta\tau$ is min time.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

EINSTEIN - MASS ENERGY RELATION

$$F = \frac{dP}{dt} = \frac{d}{dt}[mv]$$



$$\text{workdone} = \vec{F}\cdot\vec{dx} = F dx$$

$$dk = \Delta KE = \text{workdone} = \frac{dP}{dt}dx = v dP$$

$$= v d(mv)$$

$$dk = v\left[m dv + v dm\right]$$

$$dk = vm\, dv + v^2 dm$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)} \Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2$$

Differentiating.

$$2m\,dm\,c^2 - \left[2m\,dm\,v^2 + m^2 2v\,dv\right] = 0$$

$$c^2 dm - v^2 dm - mv\,dv = 0$$

$$c^2 dm = v^2 dm + mv\,dv$$

$$dk = vm\,dv + v^2 dm = c^2 dm$$

Integrate

$$\int_0^k dk = \int_{m_0}^{m} c^2 dm = c^2\left[m - m_0\right]$$

$K = c^2 m - m_0 c^2$

$K + m_0 c^2 = mc^2$

$E = mc^2$ $\quad\quad\quad\quad$ $E = K + m_0 c^2$

E = Total Energy $= mc^2$

K = Kinetic Energy

$m_0 c^2$ = rest mass energy

Kinetic energy + Restmass Energy = Total Energy

$E = mc^2 = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \cdot c^2 = k m_0 c^2$; $k = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$P = mv = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} v = k m_0 v$

$$\boxed{\begin{aligned} k^2 &= \frac{c^2}{c^2 - v^2} \\ k^2c^2 - k^2v^2 &= c^2 \\ k^2c^2 - c^2 &= k^2v^2 \end{aligned}}$$

$P^2 = k^2 m_0^2 v^2$

$P^2 = [k^2c^2 - c^2] m_0^2$

$P^2 = m_0^2 c^2 k^2 - m_0^2 c^2$

$P^2c^2 = m_0^2 c^4 k^2 - m_0^2 c^4$

$P^2c^2 = E^2 - m_0^2 c^4$

$\boxed{E^2 = m_0^2 c^4 + p^2 c^2}$ Relation b/w P & E

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gas

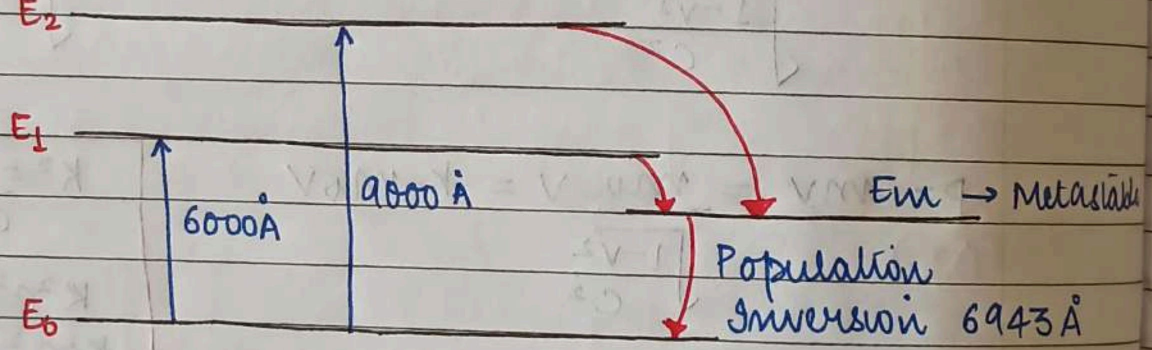
RUBY LASER AND HE-NE LASER

Al_2O_3

Al_2O_3 dopped with chromium ($0.05\% - 0.5\%$)

1) Stimulated absorption
2) Spontaneous Emission
3) Stimulated Emission
4) Population Inversion
5) Metastable State
6) Einstein coefficients

(i) Pumping
(ii) Resonance Cavity
(iii) Active center



In Metastable State :- life Time of excited state is $10^{-8} - 10^{-9}$ sec

chromium excited state is Metastable state

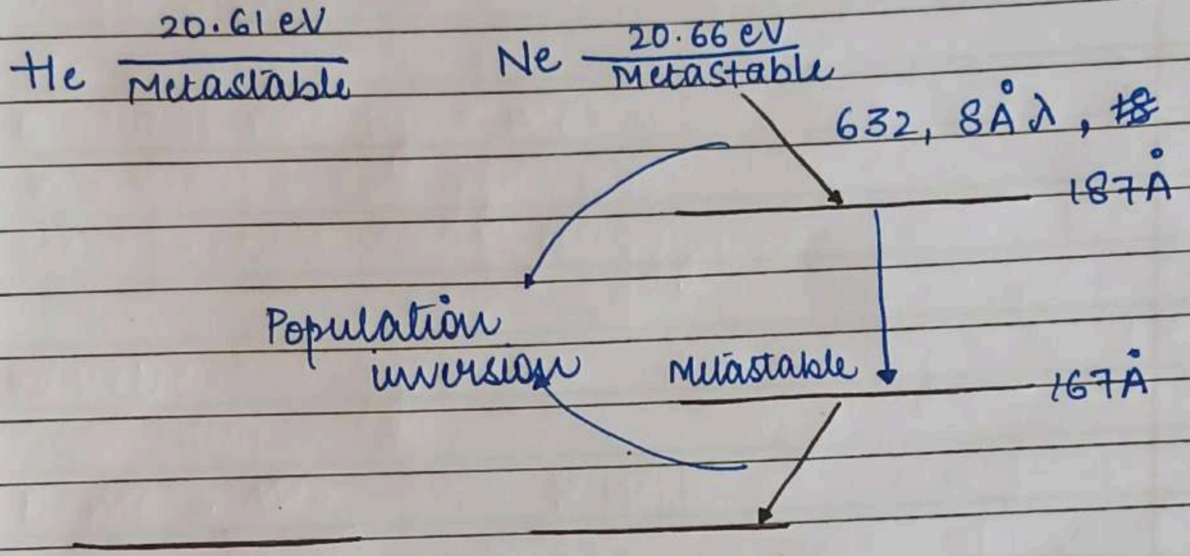
Power Supply to,

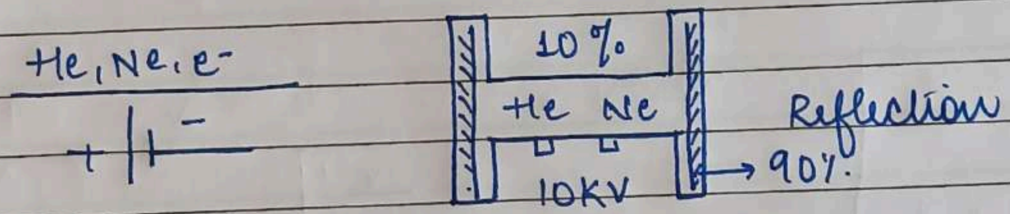
Pumping { Transfer of energy from Ground to excited state
Electrical pumping :- In He-Ne laser
optical pumping :- In Ruby laser

Active centre { chromium is active centre in Ruby ~~et~~ laser bcz it is Meta- stable state

Resonance cavity { In which volume of laser amplifies

HE-NE LASER :-





Q. Working of He-Ne Laser
Q. Diff b/w He & Ne
Q. Why combination of He & Ne is taken

Work of He :- To transfer Ne in excited state

- → Both He & He have approx same energy in Metā stable state
- → He is lighter than Ne.
- → e⁻ will discharged tube will collapse to He and then He will transfer Ne to its excited state

$$He + e^{*} \longrightarrow He^{*} + e$$
$$Ne + He^{*} \longrightarrow Ne^{*} + He$$

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TRANSFORMATION EQUATION
for Electric field & Magnetic field

$F'_x = F_x - \frac{V}{C^2} \vec{F} \cdot \vec{u}$ V :- Relative Velocity of S' wrt S

$F'_y = \frac{F_y}{K\left[1 - \frac{V u_x}{C^2}\right]}$ $F'_z = \frac{F_z}{K\left[1 - \frac{V u_x}{C^2}\right]}$

u = particle velocity in S' frame

$$\vec{F}' = q\left[\vec{E}' - \vec{V} \times \vec{B}'\right]$$

$$= q\left[\vec{E}_x \hat{i} + E'_y \hat{j} + E_z \hat{k}\right] - q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V & 0 & 0 \\ B'_x & B'_y & B'_z \end{vmatrix}$$

$$= q\left[E'_x \hat{i} + E'_y \hat{j} + E_z \hat{k}\right] - q\left[-\hat{j}(VB'_z) + \hat{k}(VB'_y)\right]$$

$$= \cancel{q\left[\vec{E}_y + VB_z\right.}$$

$$= q\left[E'_x \hat{i} + \hat{j}(E'_y + VB_z) + \hat{k}\left[E'_z - VB'_y\right]\right]$$

$$F'_y = \frac{F_y}{K\left[1 - \frac{V u_x}{C^2}\right]} = \frac{F_y}{K} = E'_y + VB_z = E_y$$